Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Monday 19 October 2020

Afternoon	Paper Reference **9MA0/31**

Mathematics

Advanced

Paper 31: Statistics

You must have: Mathematical Formulae and Statistical Tables (Green), calculator	Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

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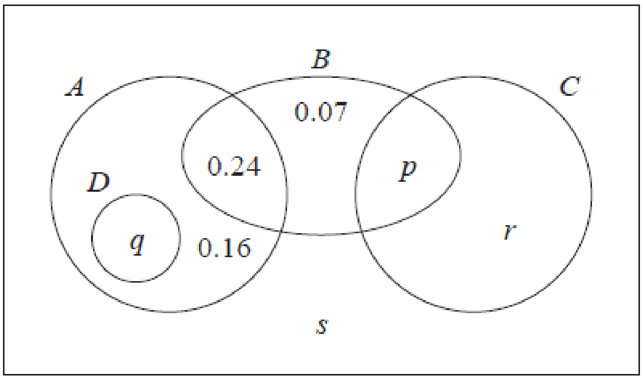
Turn over

1 The Venn diagram shows the probabilities associated with four events, A, B, C and D



(a) Write down any pair of mutually exclusive events from A, B, C and D

(1)

Given that $\mathrm{P}(B) = 0.4$

(b) find the value of p

(1)

Given also that A and B are independent

(c) find the value of q

(2)

Given further that $\mathrm{P}(B' \mid C) = 0.64$

(d) find

(i) the value of r

(ii) the value of s

(4)

(Total for Question 1 is 8 marks)

2 A random sample of 15 days is taken from the large data set for Perth in June and July 1987.

The scatter diagram in Figure 1 displays the values of two of the variables for these 15 days.

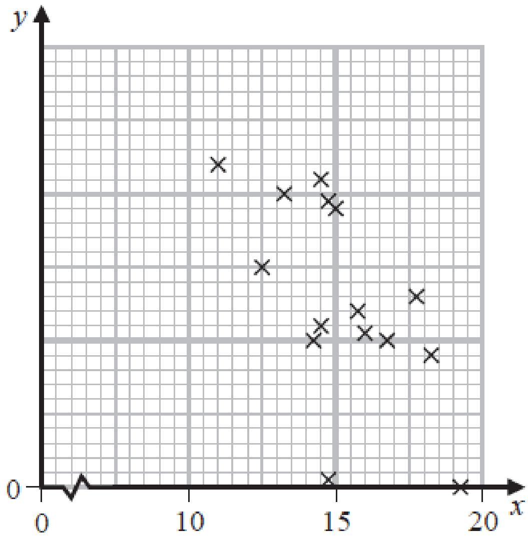


Figure 1

(a) Describe the correlation.

(1)

The variable on the x-axis is Daily Mean Temperature measured in °C.

(b) Using your knowledge of the large data set,

(i) suggest which variable is on the y-axis,

(ii) state the units that are used in the large data set for this variable.

(2)

Stav believes that there is a correlation between Daily Total Sunshine and Daily Maximum Relative Humidity at Heathrow.

He calculates the product moment correlation coefficient between these two variables for a random sample of 30 days and obtains $r = -0.377$

(c) Carry out a suitable test to investigate Stav's belief at a 5% level of significance. State clearly

- your hypotheses
- your critical value

(3)

On a random day at Heathrow the Daily Maximum Relative Humidity was 97%

(*d*) Comment on the number of hours of sunshine you would expect on that day, giving a reason for your answer.

(1)

(Total for Question 2 is 7 marks)

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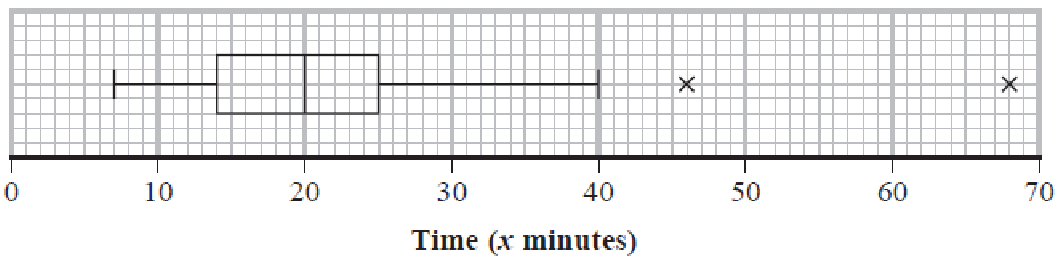
3 Each member of a group of 27 people was timed when completing a puzzle.

The time taken, x minutes, for each member of the group was recorded.

These times are summarised in the following box and whisker plot.



(a) Find the range of the times.

(1)

(b) Find the interquartile range of the times.

(1)

For these 27 people $\sum x = 607.5$ and $\sum x^2 = 17623.25$

(c) calculate the mean time taken to complete the puzzle,

(1)

(d) calculate the standard deviation of the times taken to complete the puzzle.

(2)

Taruni defines an outlier as a value more than 3 standard deviations above the mean.

(e) State how many outliers Taruni would say there are in these data, giving a reason for your answer.

(1)

Adam and Beth also completed the puzzle in a minutes and b minutes respectively, where $a > b$.

When their times are included with the data of the other 27 people

- the median time increases
- the mean time does not change

(f) Suggest a possible value for a and a possible value for b, explaining how your values satisfy the above conditions.

(3)

(*g*) Without carrying out any further calculations, explain why the standard deviation of all 29 times will be lower than your answer to part (d).

(1)

(Total for Question 3 is 10 marks)

---

4 The discrete random variable D has the following probability distribution

d	10	20	30	40	50
$\mathrm{P}(D = d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where k is a constant.

(*a*) Show that the value of k is $\frac{600}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(*b*) Find P $(D_1 + D_2 = 80)$
Give your answer to 3 significant figures.

(3)

A single observation of D is made.

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(*c*) Find the exact probability that the smallest angle of Q is more than 50°

(5)

(Total for Question 4 is 10 marks)

---

5 A health centre claims that the time a doctor spends with a patient can be modelled by a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes.

(*a*) Using this model, find the probability that the time spent with a randomly selected patient is more than 15 minutes.

(1)

Some patients complain that the mean time the doctor spends with a patient is more than 10 minutes.

The receptionist takes a random sample of 20 patients and finds that the mean time the doctor spends with a patient is 11.5 minutes.

(*b*) Stating your hypotheses clearly and using a 5% significance level, test whether or not there is evidence to support the patients' complaint.

(4)

The health centre also claims that the time a dentist spends with a patient during a routine appointment, T minutes, can be modelled by the normal distribution where $T \sim \mathrm{N}(5, 3.5^2)$

(*c*) Using this model,

(i) find the probability that a routine appointment with the dentist takes less than 2 minutes

(1)

(ii) find $\mathrm{P}\left(T < 2 \mid T > 0\right)$

(3)

(iii) hence explain why this normal distribution may not be a good model for T.

(1)

The dentist believes that she cannot complete a routine appointment in less than 2 minutes.

She suggests that the health centre should use a refined model only including values of $T > 2$

(*d*) Find the median time for a routine appointment using this new model, giving your answer correct to one decimal place.

(5)

(Total for Question 5 is 15 marks)

TOTAL FOR PAPER IS 50 MARKS

GCE A level Mathematics (9AM0) – Paper 31
Statistics

October 2020 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	A and C **or** D and B **or** D and C	B1 1.2	This mark is given for one correct pair stated (if more than one pair is stated, they must be correct)
(b)	$p = 0.4 - 0.07 - 0.24 = 0.09$	B1 1.1b	This mark is given for the correct answer only
(c)	$\text{P}(A) \times 0.4 = 0.24$ $\text{P}(A) = 0.6$	M1 1.1b	This mark is given for a correct equation for $\text{P}(A)$
	$q = 0.20$	A1 1.1b	This mark is given for the correct answer only
(d)(i)	$\text{P}(B' \mid C) = 0.64 \Rightarrow \dfrac{r}{r+p} = 0.64$ $r = 0.64r + 0.64p$ $0.36r = 0.0576$	M1 3.1a	This mark is given for the use of $\text{P}(B' \mid C) = 0.64$ leading to a correct formula in terms of r and p
	$r = 0.16$	A1 1.1b	This mark is given for the correct answer only
(d)(ii)	$0.6 + 0.07 + 0.25 + s = 1$	M1 1.1b	This mark is given for using the total probability = 1 to form a linear equation for s
	$s = 0.08$	A1 1.1b	This mark is given for the correct answer only

Question 2 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	Negative	B1 1.2	This mark is given for the correct answer only
(b)	Rainfall	B1 2.2b	This mark is given for the correct answer only
	mm	B1 1.1b	This mark is given for the correct answer only
(c)	H_0: $\rho = 0$, H_1: $\rho \neq 0$	B1 2.5	This mark is given for two correct hypotheses in terms of ρ
	Critical value: – 0.3610	M1 1.1b	This mark is given for a correct critical value found
	$r < -0.3610$ a significant result; thus there is evidence of a correlation between the Daily Total Sunshine and the Daily Maximum Relative Humidity	A1 2.2b	This mark is given for a correct conclusion given in context
	Humidity is high and there is evidence of correlation and $r < 0$ Thus we would expect the amount of sunshine to be lower than the average for Heathrow	B1 2.2b	This mark is given for a conclusion for stating referring to high humidity, $r < 0$ and a low amount of sunshine

Question 3 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$68 - 7 = 61$	B1 1.1b	This mark is given for the correct answer only
(b)	$25 - 14 = 11$	B1 1.1b	This mark is given for the correct answer only
(c)	$\bar{x} = \frac{607.5}{27} = 22.5$	B1 1.1b	This mark is given for the correct answer only
(d)	$\sigma = \sqrt{\frac{17623.25}{27} - 22.5^2}$	M1 1.1b	This mark is given for a method to calculate the standard deviation
	$= 12.1$	A1 1.1b	This mark is given for the correct answer only (to 1 decimal place)
(e)	$\mu + 3\sigma = 22.5 + (3 \times 12.1) = 58.8$ Only one outlier	B1 1.1b	This mark is given for the correct answer and reason given
(f)	The median time increases means that both values must be > 20	M1 3.1b	This mark is given for a correct statement about the values based on the median
	The mean time does not change is the same means that $a + b = 45$	M1 1.1b	This mark is given for for a correct explanation leading to an equation for $a + b$
	For example: Possible values are $b = 21$ and $a = 24$	A1 2.2b	This mark is given for a correct pair of values (a and b both > 20 and adding to 45) stated
(g)	Both values will be less than 1 standard deviation from the mean, so the standard deviation of all 29 values will be smaller	B1 2.4	This mark is given for a correct explanation

Question 4 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ $\frac{1}{600}(60k+30k+20k+15k+12k)=1$	M1 1.1b	This mark is given for a method to find the value of k
	$k=\frac{600}{137}$	A1 1.1b	The mark is given for fully correct working leading to the given answer
(b)	$D_1 = 30$, $D_2 = 50$ or $D_1 = 50$, $D_2 = 30$ or $D_1 = 40$, $D_2 = 40$	M1 2.1	This mark is given for a method to select all the relevant cases
	$\mathrm{P}(D_1 + D_2 = 80) = \frac{k}{50}\times\frac{k}{30}\times 2+\left(\frac{k}{40}\right)^2$	M1 3.4	This mark is given for a method to use the model to obtain a correct expression for the probability
	$= 0.0376$	A1 1.1b	This mark is given for the correct answer only
(c)	Angles are a, $a + d$, $a + 2d$, $a + 3d$	M1 3.1a	This mark is given for a method to find expressions for the four angles in terms of a and d
	$S_4 = 2(2a + 3d) = 360$	M1 2.1	This mark is given for using the properties of a quadrilateral with the four angles
	$2a + 3d = 180$	A1 2.2a	This mark is given for finding $2a + 3d = 180$
	Smallest angle is $a > 50$ $d = 10$ so $a = 75$ or $d = 20$ so $a = 60$	M1 3.1b	This mark is given for a method to examine the possible cases and using $d = 10$ and $d = 20$ only
	$\mathrm{P}(d = 10 \text{ or } 20) = \frac{k}{10}+\frac{k}{20}=\frac{3k}{20}=\frac{90}{137}$	A1 1.1b	This mark is given for a correct answer only

Question 5 (Total 15 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$P(x > 15) = 0.106$	B1 1.1b	This mark is given for the correct answer only
(b)	H_0: $\mu = 10$, H_1: $\mu > 10$	B1 2.5	This mark is given for two correct hypotheses in terms of μ
	$\overline{X} \sim N\left(10, \left(\frac{4}{\sqrt{20}}\right)^2\right)$	M1 3.3	This mark is given for a selection of a correct model
	$P(\overline{X} > 11.5) = 0.046766$	A1 3.4	This mark is given for using this model to find the probability
	This is significant (< 5%) so there is evidence to support the complaint	A1 2.2b	This mark is given for a correct conclusion in context
(c)(i)	$P(T < 2) = 0.196$	B1 1.1b	This mark is given for the correct answer only
(c)(ii)	$\frac{P(0 < T < 2)}{P(T > 0)} = \frac{0.119119}{0.923436}$	M1 3.4	This mark is given for a method to form a probability ratio expression
		A1 1.1b	This mark is given for a correct ratio of probabilities
	$= 0.1289955$	A1 1.1b	This mark is given for a correct answer only
(c)(iii)	The current model suggests a non-negligible probability of T values < 0, which is impossible	B1 3.5b	This mark is given for a suitable explanation of why model is not suitable based on negative T values
(d)	See over		

Question 5 (Total 15 marks) continued

Part	Working or answer an examiner might expect to see	Mark	Notes
(d)	$\mathrm{P}(T > t \mid T > 2) = 0.5$	M1 3.1b	This mark is given for a correct conditional probability statement to start the problem
	$\dfrac{\mathrm{P}(T > t)}{\mathrm{P}(T > 2)} = 0.5$	M1 1.1b	This mark is given for a correct ratio of probability expressions
	$\mathrm{P}(T > t) = 0.5 \times (1 - 0.196)$	A1 3.4	This mark is given for a correct equation for $\mathrm{P}(T > t)$
	$\mathrm{P}(T > t) = 0.402 \Rightarrow \dfrac{t - 5}{3.5} = 0.2533$ or $\mathrm{P}(t < t) = 0.5978$	M1 1.1b	This mark is given for a method to find t
	$t = 5.9$ (to 1 decimal place)	A1 1.1b	This mark is given for a correct answer only

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Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel Level 3 GCE

Tuesday 20 June 2023

Afternoon

Paper reference **9MA0/31**

Mathematics

Advanced

PAPER 31: Statistics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

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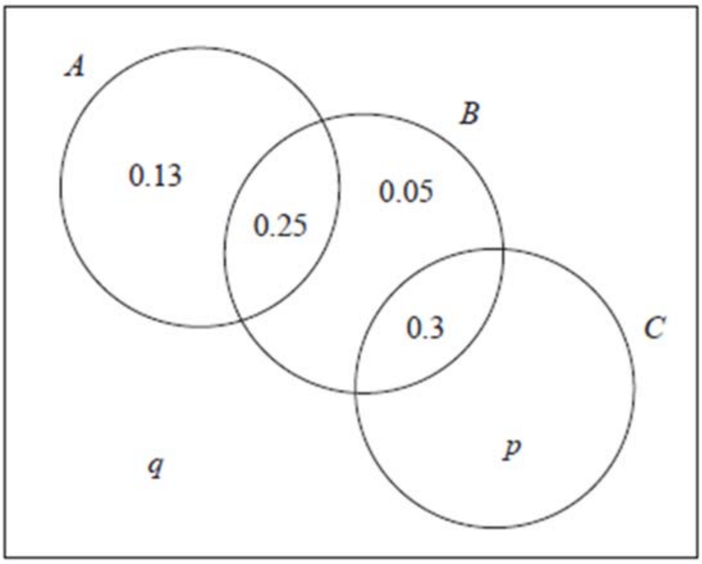
Turn over ▶

P72819A

1. The Venn diagram, where p and q are probabilities, shows the three events A, B and C and their associated probabilities.



(*a*) Find P(A)

(1)

The events B and C are independent.

(*b*) Find the value of p and the value of q

(3)

(*c*) Find P($A|B'$)

(2)

(Total for Question 1 is 6 marks)

---

2. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.

The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(*a*) State a condition needed for T to be modelled by $\mathrm{B}(40, \frac{1}{7})$

(1)

A box is selected at random.

(*b*) Using $T \sim \mathrm{B}(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(*c*) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(*d*) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

(Total for Question 2 is 9 marks)

---

3. Ben is studying the Daily Total Rainfall, x mm, in Leeming for 1987

He used all the data from the large data set and summarised the information in the following table.

x	0	0.1–0.5	0.6–1.0	1.1–1.9	2.0–4.0	4.1–6.9	7.0–12.0	12.1–20.9	21.0–32.0	tr
Frequency	55	18	18	21	17	9	9	6	2	29

(*a*) Explain how the data will need to be cleaned before Ben can start to calculate statistics such as the mean and standard deviation.

(2)

Using all 184 of these values, Ben estimates $\sum x = 390$ and $\sum x^2 = 4336$

(*b*) Calculate estimates for

(i) the mean Daily Total Rainfall,

(ii) the standard deviation of the Daily Total Rainfall.

(3)

Ben suggests using the statistic calculated in part (b)(i) to estimate the annual mean Daily Total Rainfall in Leeming for 1987

(*c*) Using your knowledge of the large data set,

(i) give a reason why these data would not be suitable,

(ii) state, giving a reason, how you would expect the estimate in part (b)(i) to differ from the actual annual mean Daily Total Rainfall in Leeming for 1987

(2)

(Total for Question 3 is 7 marks)

---

4. A study was made of adult men from region A of a country.
It was found that their heights were normally distributed with a mean of 175.4 cm and standard deviation 6.8 cm.

(a) Find the proportion of these men that are taller than 180 cm.

(1)

A student claimed that the mean height of adult men from region B of this country was different from the mean height of adult men from region A.

A random sample of 52 adult men from region B had a mean height of 177.2 cm

The student assumed that the standard deviation of heights of adult men was 6.8 cm both for region A and region B.

(b) Use a suitable test to assess the student's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

(c) Find the p-value for the test in part (b)

(1)

(Total for Question 4 is 6 marks)

---

5. Tisam is playing a game.
She uses a ball, a cup and a spinner.

The random variable X represents the number the spinner lands on when it is spun.
The probability distribution of X is given in the following table

x	20	50	80	100
$\mathrm{P}(X = x)$	a	b	c	d

where a, b, c and d are probabilities.

To play the game

- the spinner is spun to obtain a value of x
- Tisam then stands x cm from the cup and tries to throw the ball into the cup

The event S represents the event that Tisam successfully throws the ball into the cup.

To model this game Tisam assumes that

- $\mathrm{P}(S \mid \{X = x\}) = \dfrac{k}{x}$ where k is a constant
- $\mathrm{P}(S \cap \{X = x\})$ should be the same whatever value of x is obtained from the spinner

Using Tisam's model,

(*a*) show that $c = \dfrac{8}{5}b$

(2)

(*b*) find the probability distribution of X

(5)

Nav tries, a large number of times, to throw the ball into the cup from a distance of 100 cm.
He successfully gets the ball in the cup 30% of the time.

(*c*) State, giving a reason, why Tisam's model of this game is not suitable to describe Nav playing the game for all values of X

(1)

(Total for Question 5 is 8 marks)

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6. A medical researcher is studying the number of hours, T, a patient stays in hospital following a particular operation.

The histogram on the next page summarises the results for a random sample of 90 patients.

(*a*) Use the histogram to estimate $P(10 < T < 30)$

(2)

For these 90 patients the time spent in hospital following the operation had

- a mean of 14.9 hours
- a standard deviation of 9.3 hours

Tomas suggests that T can be modelled by $N(14.9, 9.3^2)$

(*b*) With reference to the histogram, state, giving a reason, whether or not Tomas' model could be suitable.

(1)

Xiang suggests that the frequency polygon based on this histogram could be modelled by a curve with equation

$$y = kxe^{-x} \qquad 0 \leq x \leq 4$$

where

- x is measured in **tens of hours**
- k is a constant

(*c*) Use algebraic integration to show that

$$\int_0^n xe^{-x}\,dx = 1 - (n+1)e^{-n}$$

(4)

(*d*) Show that, for Xiang's model, $k = 99$ to the nearest integer.

(3)

(*e*) Estimate $P(10 < T < 30)$ using

(i) Tomas' model of $T \sim N(14.9, 9.3^2)$

(1)

(ii) Xiang's curve with equation $y = 99xe^{-x}$ and the answer to part (*c*)

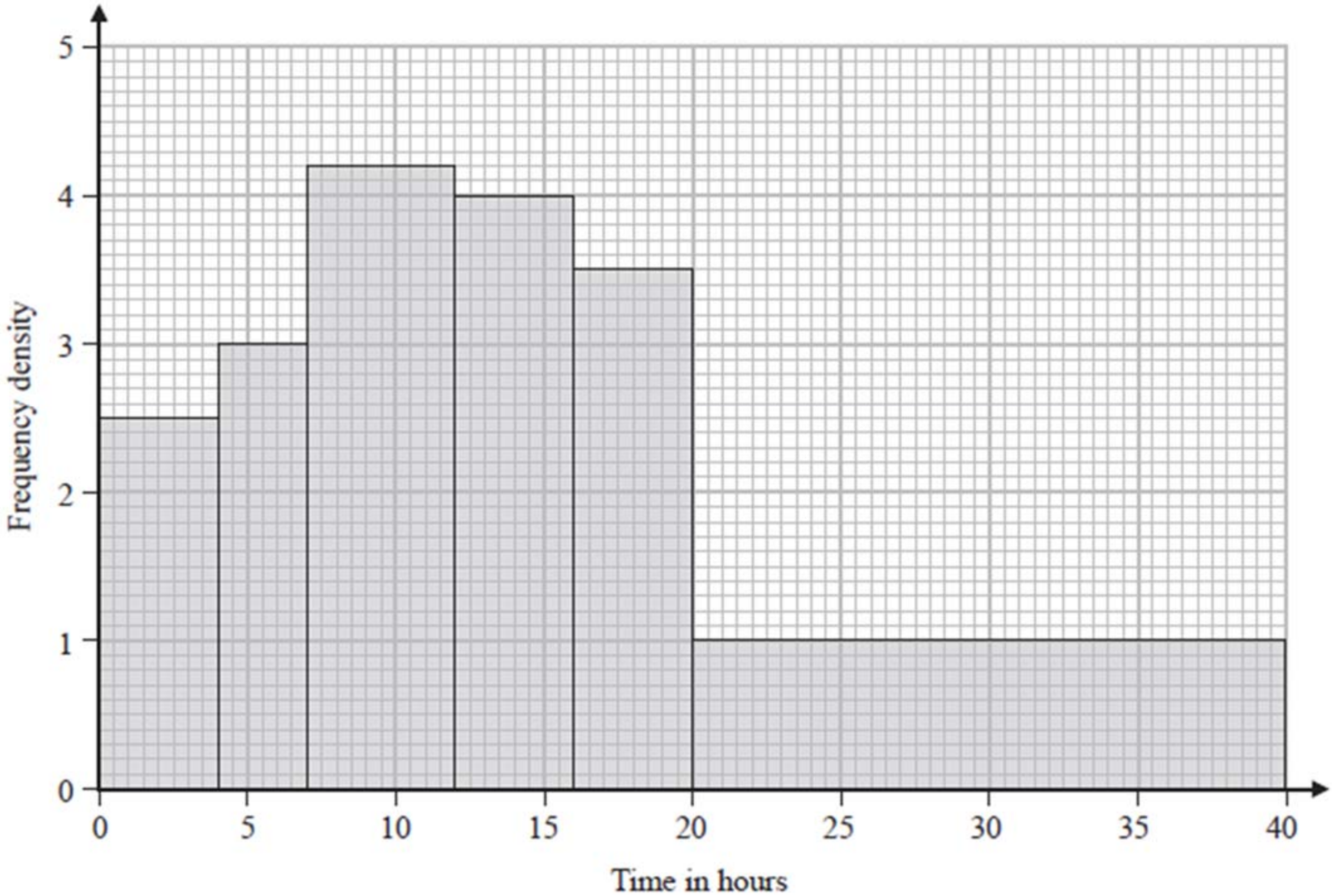
(2)

The researcher decides to use Xiang's curve to model $P(a < T < b)$

(*f*) State one limitation of Xiang's model.

(1)

Question 6 continued



(Total for Question 6 is 14 marks)

TOTAL FOR STATISTICS IS 50 MARKS

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Pearson Edexcel Level 3 GCE

Tuesday 20 June 2023

Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced
PAPER 32: Mechanics

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Total Marks

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- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

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Turn over

1. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of 3.2 m s^{-2}

Find

(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

(Total for Question 1 is 3 marks)

2.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at 1.4 m s^{-2}

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

(Total for Question 2 is 4 marks)

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3. At time t seconds, where $t \geq 0$, a particle P has velocity $\mathbf{v}$ m s^{-1} where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$

(3)

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$

(2)

(c) the acceleration of P at time t seconds

(2)

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

(2)

(Total for Question 3 is 9 marks)

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4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})$ m s^{-2}

At time $t = 0$, P passes through the point A.

At time $t = 5$ s, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})$ m s^{-1}

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})$ m.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})$ m.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

(Total for Question 4 is 10 marks)

5.

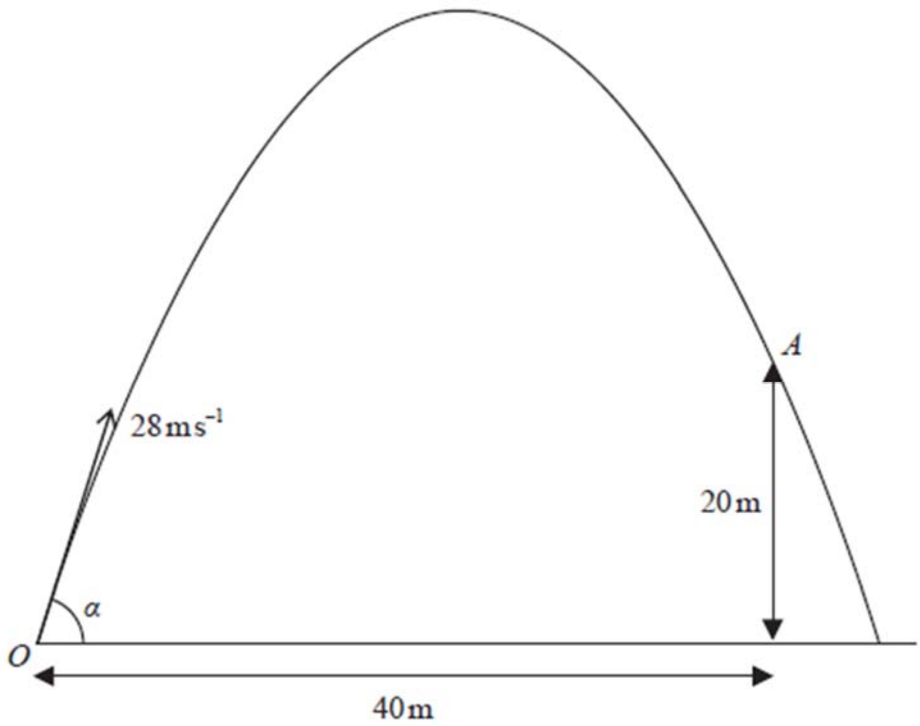

Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

(Total for Question 5 is 11 marks)

6.

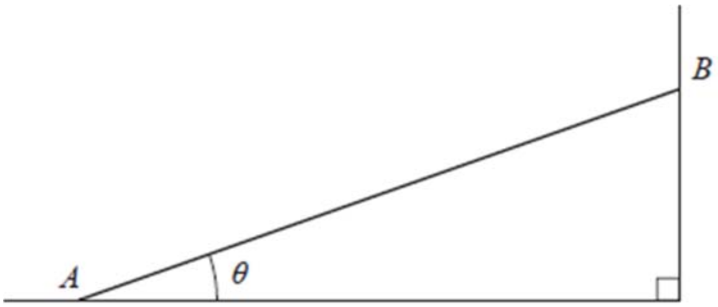

Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(*a*) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (*b*), (*c*) and (*d*).

(*b*) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(*c*) find the value of μ

(5)

(*d*) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(*e*) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

(Total for Question 6 is 13 marks)

TOTAL FOR MECHANICS IS 50 MARKS