Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number | Candidate Number

Practice Paper 1

Advanced
Paper 3: Statistics and Mechanics

Friday 15 June 2018 – Afternoon
Time: 2 hours

Paper Reference
9MA0/03

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- The total mark for this part of the examination is 100. There are 11 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

SECTION A: STATISTICS

1. An electrical engineer makes components for computer systems. She claims that the components last longer than 500 hours on 52% of occasions.

In a random sample of 40 of the components, X last longer than 500 hours.

(a) Find $P(X > 22)$.

(1)

(b) Write down two conditions under which the normal approximation may be used as an approximation to the binomial distribution.

(2)

A random sample of 250 components was taken and 120 lasted longer than 500 hours.

(c) Assuming the engineer's claim to be correct, use a normal approximation to find the probability that 120 or fewer components last longer than 500 hours.

(3)

(d) Using your answer to part **c**, comment on the engineer's claim.

(1)

(Total for Question 1 is 7 marks)

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2. $$P(A) = 0.4,\ P(B) = 0.55 \text{ and } P(C) = 0.26.$$

Given that $P(A \cap B) = 0.2$, that events A and C are mutually exclusive and that events B and C are statistically independent,

(a) Draw a Venn diagram to illustrate events A, B and C.

(5)

(b) Show that events A and B are not statistically independent.

(2)

(c) Find $P(A|B9)$.

(2)

(d) Find $P(C\,|(A \cap B)9)$.

(2)

(Total for Question 2 is 11 marks)

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3. The daily mean air temperature, t °C, is recorded in Perth for the month of October.

t	$12 \leq t < 15$	$15 \leq t < 18$	$18 \leq t < 20$	$20 \leq t < 22$	$22 \leq t < 26$
f	2	6	11	7	5

(a) State, with a reason, whether t is a discrete or continuous variable.

(1)

(b) Use your calculator to find estimates for the mean and standard deviation of the temperatures.

(2)

(c) Give two reasons why a histogram could be used to display this data.

(2)

(d) Use linear interpolation to find the 10th to 90th interpercentile range.

(3)

A meteorologist believes that there is a positive correlation between the daily mean air temperature and the number of hours of sunshine. She takes a random sample of 8 days from the data set above and finds that the product moment correlation coefficient is 0.612.

(e) Stating your hypotheses clearly, test at the 5% level of significance, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

(Total for Question 3 is 11 marks)

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4. An industrial chemical process produces an amount of a substance, q grams, dependent on the temperature, t °C applied. The table below shows the outcomes of five experiments.

t	10	20	32	41	57
q	160	700	2000	3300	6400

A chemist believes that the relationship between the variables can be modelled by an equation of the form $q = kt^n$, where k and n are constants to be determined. The data are coded using $x = \log t$ and $y = \log q$. The product moment correlation coefficient between x and y is found to be 0.9998.

(a) State with a reason whether this value supports the suggested model.

(1)

(b) Given that the equation of the regression line of y on x is $y = 0.0761 + 2.1317x$, find the value of k and the value of n.

(3)

(c) Explain, giving a reason, whether it would be sensible to use this model to predict the amount of substance produced when $t = 85$ °C.

(1)

(Total for Question 4 is 5 marks)

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5. The weights of cats in a particular town are normally distributed. A cat that weighs between 3.5 kg and 4.6 kg is said to be of 'standard' weight.

Given that 2.5% of cats weigh less than 3.416 kg and 5% of cats weigh greater than 4.858 kg,

(a) find the proportion of cats that are of standard weight.

(6)

15 cats are chosen at random.

(b) Find the probability that at least 10 of these cats are of standard weight.

(2)

In a second town, the weights of cats are also normally distributed with standard deviation 0.51 kg. A random sample of 12 cats was taken and the sample mean was 4.73 kg.

(c) Test, at the 10% level of significance, whether or not the mean weight of all the cats in the town is different from 4.5 kg. State your hypotheses clearly.

(4)

(Total for Question 5 is 12 marks)

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6. Jemima plays two games of tennis. The probability that she wins the first game is 0.62. If she wins the first game, the probability that she wins the second is 0.75. If she loses the first game, the probability that she wins the second is 0.45.

Find the probability that she wins both games given that she wins the second game.

(Total for Question 6 is 4 marks)

TOTAL FOR STATISTICS IS 50 MARKS

SECTION B: MECHANICS

7. At time t seconds, where $t > 0$, a particle P moves such that its velocity, $\mathbf{v}$ m s^{-1}, is given by $\mathbf{v} = (2 - 6t^2)\mathbf{i} - t\mathbf{j}$.

When $t = 1$, the displacement of the particle from a fixed origin O is $5\mathbf{i}$ m.

Find the distance of the particle from O when $t = 3$ seconds, giving your answer to 3 significant figures.

(Total for Question 7 is 7 marks)

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8. An arrow is fired from horizontal ground with an initial speed of 100 m s^{-1} at an angle of 30° above the horizontal.

By modelling the arrow as a particle, work out

(a) the time taken for the arrow to hit the ground,

(3)

(b) the maximum height of the arrow,

(3)

(c) the speed of the arrow after 3 seconds.

(6)

(Total for Question 8 is 12 marks)

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9. [*In this question* **i** *and* **j** *are the unit vectors due east and north respectively.*]

A cyclist makes a journey between two points A and B**.** At time $t = 0$ s the cyclist is moving due east at 2 m s^{-1}. The cyclist is modelled as a particle. Relative to A, the position vector of the cyclist at time t seconds is $\mathbf{r}$ metres.

Given that the acceleration of the cyclist is constant and equal to $0.2\mathbf{i} - 0.8\mathbf{j}$ m s^{-2}, find

(a) the position vector of the cyclist after 10 seconds,

(2)

(b) the distance of the cyclist from A after 10 seconds.

(2)

After 10 seconds the cyclist stops accelerating and travels due east at a constant speed of 5 m s^{-1}.

(c) Find the value of t when the cyclist is directly south-east of A.

(2)

After a further 30 s the cyclist reached point B.

(d) Work out the bearing of B from A to the nearest degree.

(2)

(Total for Question 9 is 8 marks)

10.

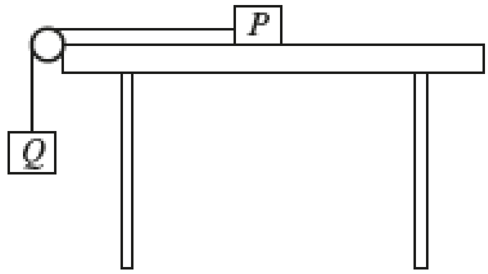

Figure 1

Two particles P and Q, of masses 3 kg and 2 kg respectively, are attached to the ends of a light inextensible string. P lies on a rough horizontal table. The string passes over a small smooth pulley fixed on the edge of the table. Q hangs freely below the pulley, as shown in Figure 1.

The coefficient of friction between P and the table is μ. The particles are released from rest with the string taut. Immediately after release, P accelerates at a rate of 0.5 m s^{-2}.

(a) Find the tension in the string immediately after the particles begin to move.

(3)

(b) Show that $\mu = 0.582$, giving your answer to 3 significant figures.

(3)

After two seconds the string breaks.

(c) Assuming that P remains on the table, calculate how long it takes P to come to rest.

(6)

(d) State how you have used the information that the string is inextensible in your calculations.

(1)

(Total for Question 10 is 13 marks)

11.

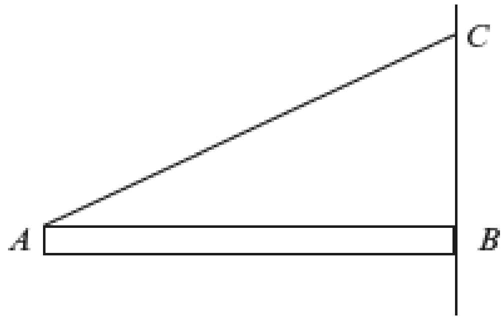

Figure 2

A uniform rod AB of length l m and mass m kg rests with one end touching a rough vertical wall at B. The rod is kept horizontal by a light inextensible string AC where C lies on the wall directly above B.

The plane ABC is perpendicular to the wall and $\angle BAC$ is α, where $\tan \alpha = \frac{5}{12}$.

(a) Show that the tension in the string is $\frac{13}{10} mg$ N.

(4)

(b) Calculate the coefficient of friction between the rod and the wall.

(6)

(Total for Question 11 is 10 marks)

TOTAL FOR MECHANICS IS 50 MARKS

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Exam-style practice: Paper 3, Section A: Statistics

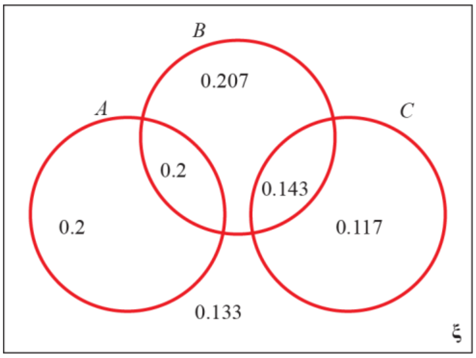
1 a Use the cumulative binomial distribution tables, with n = 40 and p = 0.52. Then $P(X \geqslant 22) = 1 - P(X \leqslant 21) = 1 - 0.5867 = 0.4133$ (4 s.f.).

b In order for the normal approximation to be used as an approximation to the binomial distribution the two conditions are: (i) n is large (>50); and (ii) p is close to 0.5.

c The two conditions for the normal approximation to be a valid approximation are satisfied. $\mu = np = 250 \times 0.52 = 130$ and $\sigma = \sqrt{np(1-p)} = \sqrt{130 \times 0.48} = \sqrt{62.4} = 7.90$ (3 s.f.). Therefore $B(250, 0.52) \approx N(130, 7 \cdot 9^2)$ so that $P(B \leqslant 120) \approx P(N \leqslant 120.5) = 0.1146$ (4 s.f.).

d If the engineer's claim is true, then the observed result had a less than 12% chance of occurring. This would mean that there would be insufficient evidence to reject her claim with a two-tailed hypothesis test at the 10% level. Though it does provide some doubt as to the validity of her claim.

2 a Since A and C are mutually exclusive, $P(A \cap C) = 0$ and their intersection need not be represented on the Venn diagram. Since B and C are independent, $P(B \cap C) = P(B) \times P(C) = 0.55 \times 0.26 = 0.143$. Using the remaining information in the question allows for the other regions to be labelled. Therefore the completed Venn diagram should be:



b $P(A) \times P(B) = 0.4 \times 0.55 = 0.22 \neq 0.2 = P(A \cap B)$ and so the events are not independent.

c $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{0.2}{0.2 + 0.117 + 0.133} = \dfrac{0.2}{0.45} = 0.444$ (3 s.f.)

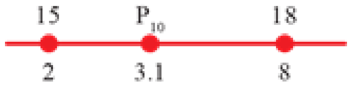
d $P(C \mid (A \cap B)') = \dfrac{P(C \cap (A \cap B)')}{P((A \cap B)')} = \dfrac{P(C)}{1 - P(A \cap B)} = \dfrac{0.26}{0.8} = 0.325$

3 a The variable t is continuous, since it can take any value between 12 and 26 (in degrees Celsius).

b Estimated mean 19.419; estimated standard deviation 2.814 (3 d.p.).

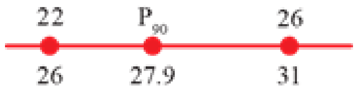
c Temperature is continuous and the data were given in a grouped frequency table.

3 d The 10th percentile is $\frac{31}{10} = 3.1$th value. Using linear interpolation:



$$\frac{P_{10}-15}{18-15}=\frac{3.1-2}{8-2}\Rightarrow\frac{P_{10}-15}{3}=\frac{1.1}{6}\Rightarrow P_{10}=3\times\frac{1.1}{6}+15=0.55+15=15.5$$

The 90th percentile is $\frac{9\times 31}{10} = 27.9$th value. Using linear interpolation:



$$\frac{P_{90}-22}{26-22}=\frac{27.9-26}{31-26}\Rightarrow\frac{P_{90}-22}{4}=\frac{1.9}{5}\Rightarrow P_{90}=4\times\frac{1.9}{5}+22=1.52+22=23.52$$

Therefore the 10th to 90th interpercentile range is $23.52-15.55=7.97$.

e Since the meteorologist believes that there is positive correlation, the hypotheses are
$H_0 : \rho = 0$
$H_1 : \rho > 0$
The sample size is 8, and so the critical value (for a one-tailed test) is 0.6215.
Since $r = 0.612 < 0.6215$, there is not sufficient evidence to reject H_0, and so there is not sufficient evidence, at the 5% significance level, to say that there is a positive correlation between the daily mean air temperature and the number of hours of sunshine.

4 a The value of 0.9998 is very close to 1, indicating that the plot of x against y is very close to being a linear relationship, and so the data should be well-modelled by an equation of the form $q = kt^n$.

b Rearranging the equation
$y = 0.07601 + 2.1317x$
$\Rightarrow \log q = 0.07601 + 2.1317\log t$
$\Rightarrow q = 10^{0.07601+2.1317\log t} = 10^{0.07601}\times 10^{2.1317\log t}$
$\Rightarrow q = 10^{0.07601}\times 10^{\log t^{2.1317}} = 10^{0.07601}\times t^{2.1317}$
Therefore $k = 10^{0.0761} = 1.19$ (3 s.f.) and $n = 2.1317$.

c It would not be sensible to use the model to predict the amount of substance produced when $t = 85^\circ\text{C}$, since this is considerably outside the range of the provided data (extrapolation).

5 a $P(Z < a) = 0.025 \Rightarrow a = -1.96$ and $P(Z > a) = 0.05 \Rightarrow a = 1.645$. Therefore, for the given distribution, $\frac{3.416-\mu}{\sigma} = -1.96$ and $\frac{4.858-\mu}{\sigma} = 1.645$. Rearranging these equations:
$\frac{3.416-\mu}{\sigma} = -1.96 \Rightarrow 3.416 - \mu = -1.96\sigma$ and $\frac{4.858-\mu}{\sigma} = 1.645 \Rightarrow 4.858 - \mu = 1.645\sigma$.
Now subtract the second equation from the first to obtain:
$4.858 - \mu - (3.416 - \mu) = 1.645\sigma - (-1.96\sigma) \Rightarrow 1.442 = 3.605\sigma \Rightarrow \sigma = 0.4$ and so, using the first equation, $3.416 - \mu = -1.96 \times 0.4 \Rightarrow \mu = 3.416 + 0.784 = 4.2$. Using these values within the normal distribution, $P(3.5 < X < 4.6) = P(4.6) - P(3.5) \ = 0.84134 - 0.04006 = 0.8013$ (4 s.f.) of the cats will be of the standard weight.

b Using the binomial distribution, $P(B \geqslant 10) = 1 - P(B \leqslant 9) = 1 - 0.0594 = 0.9406$ (4 s.f.).

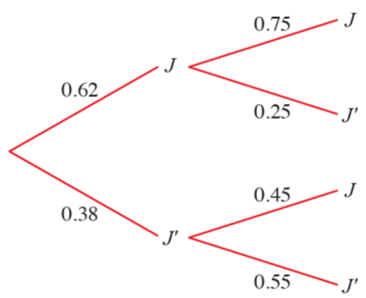
c Assume the mean is 4.5kg and standard deviation is 0.51. Then the sample $\bar{X}$ should be normally distributed with $\bar{X} \sim N\left(4.5, \frac{0.51^2}{12}\right)$. The hypothesis test should determine whether it is statistically significant, at the 10% level, that the mean is not 4.5kg. Therefore the test should be 2-tailed with

$H_0 : \mu = 4.5$
$H_1 : \mu \neq 4.5$

The critical region therefore consists of values greater than a where $P(\bar{X} > a) = 0.05$ and so $a = 4.742$ (4 s.f.) and values less than b where $P(\bar{X} < b) = 0.05$ and so $b = 4.258$ (4 s.f.).

Since the observed mean is 4.73 and 4.73 < a = 4.742, there is not enough evidence, at the 10% significance level, to reject H_0 i.e. there is not sufficient evidence to say, at the 10% level, that the mean weight of all cats in the town is different from 4.5kg.

6 It is first worth displaying the information in a tree diagram. Let J denote the event that Jemima wins a game of tennis and J' be the event that Jemima loses a game of tennis. Since Jemima either wins or loses each game of tennis, $P(J) + P(J') = 1$. This allows for the other probabilities on the tree diagram to be filled in. Therefore the completed tree diagram should be:



The required probability is then:
$P(\text{wins both games} \mid \text{wins second game})$

$$= \frac{P(\text{wins both games})}{P(\text{wins second game})} = \frac{0.62 \times 0.75}{0.62 \times 0.75 + 0.38 \times 0.45} = \frac{0.465}{0.465 + 0.171} = 0.731 \text{ (3 s.f.)}$$

Exam-style practice: Paper 3, Section B: Mechanics

7 $\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$

$$= \int (2-6t^2)\mathbf{i} - t\mathbf{j}\,\mathrm{d}t$$

$$= \left(2t - \frac{6}{3}t^3\right)\mathbf{i} - \frac{t^2}{2}\mathbf{j} + c$$

At $t = 1$ s, $\mathbf{r} = 5\mathbf{i}$ m $\Rightarrow 5\mathbf{i} = (2-2)\mathbf{i} - \frac{1}{2}\mathbf{j} + c$

$$c = 5\mathbf{i} + \frac{1}{2}\mathbf{j}$$

$$\therefore \mathbf{r} = \left(2t - 2t^3 + 5\right)\mathbf{i} + \frac{1}{2}\left(1 - t^2\right)\mathbf{j}$$

When $t = 3$ s,

$$\mathbf{r} = (6 - 54 + 5)\mathbf{i} + \frac{1}{2}(1-9)\mathbf{j}$$

$$\mathbf{r} = -43\mathbf{i} - 4\mathbf{j}$$

$$s = |\mathbf{r}| = \sqrt{43^2 + 4^2} = 43.185...$$

At $t = 3$ s, P is 43.2 m from O (3 s.f.).

8 $R(\rightarrow)$: $u_x = 100\cos 30^\circ = 50\sqrt{3}$

$R(\uparrow)$: $u_y = 100\cos 30^\circ = 50$

a $R(\uparrow)$: $u_y = 50$ ms^{-1}, s = 0 m, $a = g = -9.8$ ms^{-2}, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 50t - 4.9t^2$$

$$4.9t^2 = 50t$$

The solution $t = 0$ corresponds to the time the arrow is fired and can therefore be ignored.

$$\therefore t = \frac{50}{4.9} = 10.204...$$

The arrow reaches the ground after 10.2 s (3 s.f.).

b At maximum height, $v_y = 0$

$R(\uparrow)$: $u_y = 50$ ms^{-1}, $v_y = 0$ m, $a = g = -9.8$ ms^{-2}, $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 50^2 - 19.6s$$

$$19.6s = 2500$$

$$s = \frac{2500}{19.6} = 127.55...$$

The maximum height reached by the arrow is 128 m (3s.f.).

8 c At $t = 3$ s,

$R(\rightarrow)$: $v_x = u_x = 50\sqrt{3}$ ms^{-1} since horizontal speed remains constant.

$R(\uparrow)$: $u_y = 50$ ms^{-1}, $t = 3$ s, $a = g = -9.8$ ms^{-2}, $v_y = ?$

$$v = u + at$$

$$v_y = 50 - (3 \times 9.8) = 20.6$$

The speed at $t = 3$ s is given by:

$$v^2 = v_x^2 + v_y^2$$

$$v^2 = \left(50\sqrt{3}\right)^2 + \left(20.6\right)^2$$

$$v = \sqrt{7500 + 424.36} = 89.018...$$

The speed of the arrow after 3 s is 89.0 ms^{-1} (3 s.f).

9 a $\mathbf{u} = 2\mathbf{i}$ ms^{-1}, $t = 10$ s, $\mathbf{a} = 0.2\mathbf{i} - 0.8\mathbf{j}$ ms^{-2}, $\mathbf{r} = ?$

$$\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = 20\mathbf{i} + \frac{100}{2}(0.2\mathbf{i} - 0.8\mathbf{j})$$

$$\mathbf{r} = 20\mathbf{i} + 10\mathbf{i} - 40\mathbf{j}$$

After 10 s, the position vector of the cyclist is $(30\mathbf{i} - 40\mathbf{j})$ m.

b $s = |\mathbf{r}|$

$$s = \sqrt{30^2 + 40^2} = 50$$

After 10 s, the cyclist is 50 m from A.

c For t >10 s, $\mathbf{v} = 5\mathbf{i}$ ms^{-1} and $\mathbf{a} = 0$

The position vector is now given by:

$$\mathbf{r} = (30\mathbf{i} - 40\mathbf{j}) + \mathbf{v}(t - 10)\mathbf{i}$$

$$\mathbf{r} = 30\mathbf{i} - 40\mathbf{j} + 5(t - 10)\mathbf{i}$$

$$\mathbf{r} = (5t - 20)\mathbf{i} - 40\mathbf{j}$$

The cyclist will be south-east of A when the coefficient of $\mathbf{i}$ is positive and coefficient of $\mathbf{j}$ is negative, but both have equal magnitude.

$$5t - 20 = 40$$

$$5t = 60$$

$$t = \frac{60}{5} = 12$$

The cyclist is directly south-east of A after 12 s.

9 d First, work out the position vector of B from A:

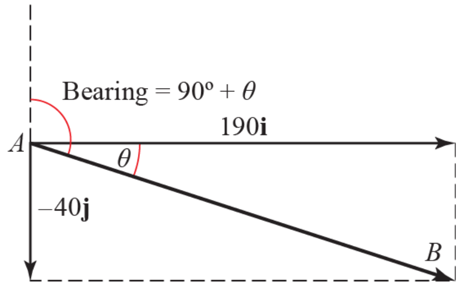
$\mathbf{r} = (5t - 20)\mathbf{i} - 40\mathbf{j}$

Cyclist reaches B when $t = 12 + 30 = 42$ s

$\mathbf{r} = ((5\times 42) - 20)\mathbf{i} - 40\mathbf{j}$

$\mathbf{r} = 190\mathbf{i} - 40\mathbf{j}$

Let θ be the acute angle between the horizontal and B (as shown in the diagram).



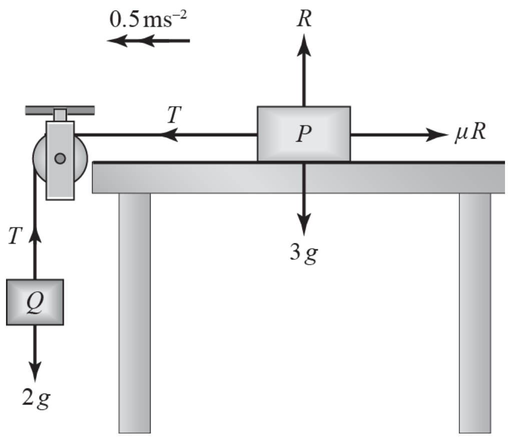
Then $\tan\theta = \dfrac{40}{190}$

$\theta = 11.888...$

To the nearest degree, the bearing of B from A is 90 +12 = 102°.

10 a Considering Q and using Newton's second law of motion:

$a = 0.5$ ms^{-2}, $m = 2$ kg



$F = ma$

$2g - T = 2\times 0.5$

$T = (2\times 9.8) - 1 = 18.6$

The tension in the string immediately after the particles begin to move is 18.6 N.

b Considering P:

Resolving vertically $\Rightarrow R = 3g$

Resolving horizontally and using Newton's second law of motion with $a = 0.5$ ms^{-2} and $m = 3$ kg:

$T - \mu R = 3\times 0.5$

$3\mu g = T - 1.5$

$\mu = \dfrac{18.6 - 1.5}{3\times 9.8} = 0.58163...$

The coefficient of friction is 0.582 (3 s.f.), as required.

10 c Consider P before string breaks: u = 0 ms^{-1}, t = 2 s, a = 0.5 ms^{-2}, v = ?

$$v = u + at$$
$$v = 0 + (0.5 \times 2) = 1$$

After string breaks, the only force acting on P is a frictional force of magnitude $F = \mu R = 3\mu g$

Using Newton's Second Law for P,

$$F = ma$$
$$3\mu g = 3a$$
$$a = \mu g$$
$$a = 9.8 \times 0.58163...$$
$$= 5.7$$

The acceleration is in the opposite direction to the motion of P, hence

u = 1 ms^{-1}, v = 0 ms^{-1}, a = −0.5 ms^{-2}, t = ?

$$v = u + at$$
$$0 = 1 - 5.7t$$
$$t = \frac{1}{5.7} = 0.17543...$$

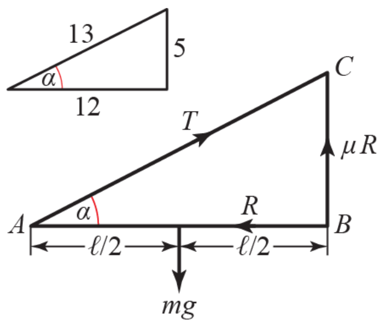
P takes 0.175 s (3 s.f.) to come to rest.

d The information that the string is inextensible has been used in assuming that the tension is the same in all parts of the string and that the acceleration of P and Q are identical while they are connected.

11 a The rod is in equilibrium so resultant force and moment are both zero.

$\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$

Taking moments about B:



$$mg\frac{l}{2} = (T\sin\alpha) \times l$$
$$T = \frac{mg}{2\sin\alpha}$$
$$T = \frac{mg}{2 \times \frac{5}{13}} = \frac{13mg}{10} \quad \text{as required.}$$

b Resolving horizontally:

$$R = T\cos\alpha$$
$$R = \frac{13mg}{10} \times \frac{12}{13} = \frac{6mg}{5}$$

Resolving vertically:

$$T\sin\alpha + \mu R = mg$$
$$\left(\frac{13mg}{10} \times \frac{5}{13}\right) + \mu\frac{6mg}{5} = mg$$
$$\frac{6}{5}\mu = 1 - \frac{1}{2}$$
$$\mu = \frac{5}{12}$$

The coefficient of friction between the rod and the wall is $\frac{5}{12}$.

Pearson Edexcel Level 3

# GCE Mathematics ## Advanced Level ## Paper 3: Statistics & Mechanics	
Practice Set 2 **Time: 2 hours**	**Paper Reference(s)** **9MA0/03**
You must have: **Mathematical Formulae and Statistical Tables, calculator**	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this paper. The total is 100.
- For each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

SECTION A: STATISTICS
Answer ALL questions.

1. An engineer believes that there is a relationship between the CO_2 emissions and fuel consumption for cars.

A random sample of 40 different car models (old and new) was taken and the CO_2 emission figures, e grams per kilometre, and fuel consumption, f miles per gallon, were recorded, as shown in Figure 1. The engineer calculates the product moment correlation coefficient for the 40 cars and obtains $r = -0.803$.

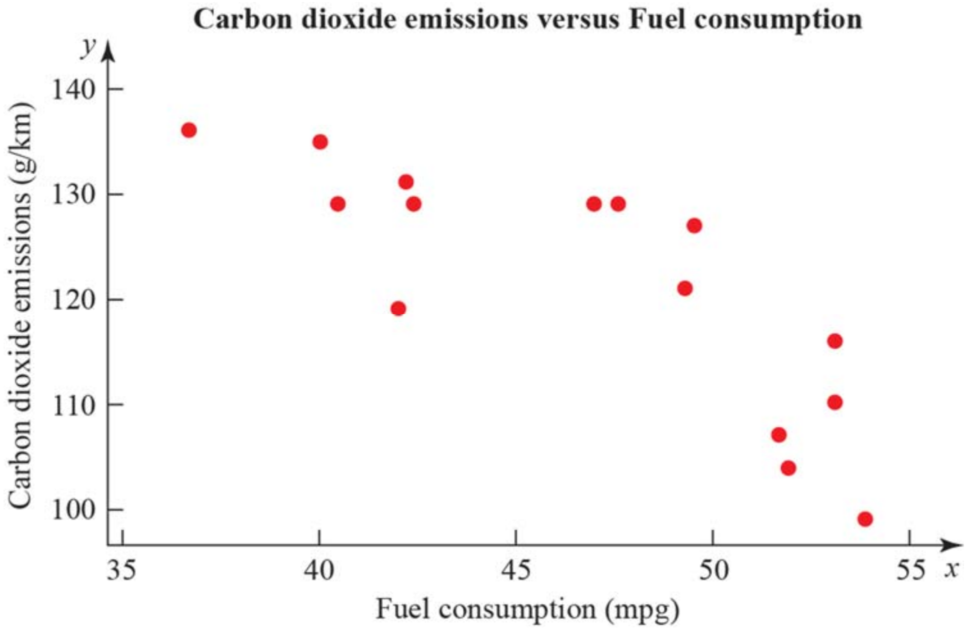


Figure 1

(a) State what is measured by the product moment correlation coefficient.
(1)

(b) State, with a reason, whether a linear regression model based on these data is reliable or not for a car when the fuel consumption is 60 mpg.
(1)

(c) For the linear regression model $e = 198 - 1.71 \times f$ write down the explanatory variable.
(1)

(d) State the definition of a hypothesis test.
(1)

(e) Test at 1% significance level whether or not the product moment correlation coefficient for CO_2 emissions and fuel consumption is less than zero. State your hypotheses clearly.
(3)

(Total 7 marks)

2. A mechanic carried out a survey on the defects of cars he was servicing. He found that the probability of a car needing a new tyre is 0.33 and that a car needing a new tyre has a probability of 0.7 of needing tracking. A car not needing a new tyre has a probability of 0.04 of needing tracking.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly chosen car has exactly one of the two defects, needing a new tyre or needing tracking.

(2)

The mechanic also finds that cars need new brake pads with probability 0.35 and that this is independent of needing new tyres or tracking. A car is chosen at random.

(c) Find the probability that the car has at least one of these three defects.

(2)

(d) What advice would you give to motorists?

(1)

(Total 8 marks)

3. The summary statistics and histogram (Figure 2) are an extract from statistical software output for the distribution of the daily mean pressure for Beijing, May to August (inclusive) 2015.

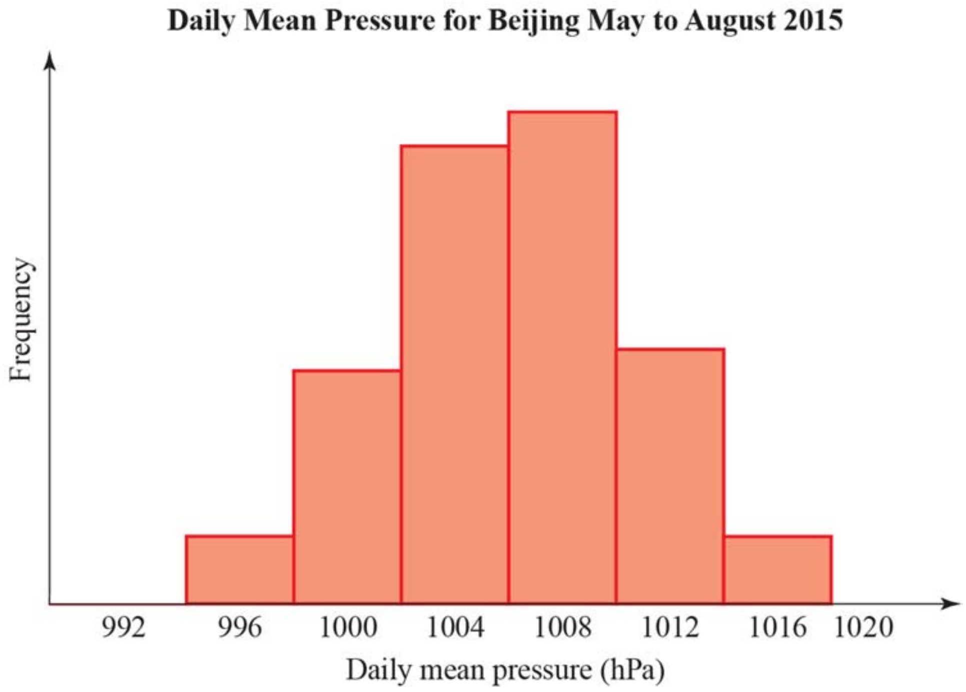


Figure 2

Variable	N	Mean	Standard deviation	Q_1	Q_2	Q_3
Daily Mean Pressure	123	1006	4.4	1003	1006	1010

(a) Explain why it is reasonable to model the daily mean pressure for Beijing, during May to August using a normal distribution.

(1)

The distribution for the daily mean pressure for Beijing, May to August 2015, X, can be modelled by a normal distribution.

Daily mean pressure (hPa)	**Suggests**
Above 1013	Good weather
Between 1013 and 1000	Fair weather
Less than 1000	Poor or bad weather
Less than 980	Hurricane

(b) Based on the statistical output and the information in the table above, what is the chance of poor or bad weather in Beijing during May to August?

(2)

(c) Although very unlikely, based on the model in part **a**, give a reason why we cannot say there is no chance of a hurricane in Beijing during May to August.

(1)

The distribution for daily mean pressure for Jacksonville during May to August can also be considered normally distributed with mean 1017 hPa and standard deviation 3.26 hPa. A student claims that you can depend on better weather in Jacksonville than in Beijing during May to August.

(d) State, giving reasons, whether the information in this question supports this claim.

(4)

(Total 8 marks)

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4. To investigate if there is a correlation between daily mean pressure (hPa) and daily mean wind speed (kn) the location Hurn 2015 was randomly selected from:

Camborne 2015	Camborne 1987
Hurn 2015	Hurn 1987
Leuchars 2015	Leuchars 1987
Leeming 2015	Leeming 1987
Heathrow 2015	Heathrow 1987.

(Source: Pearson Edexcel GCE AS and A Level Mathematics data set.)

The statistical software output for these data is shown in Figure 3 below.

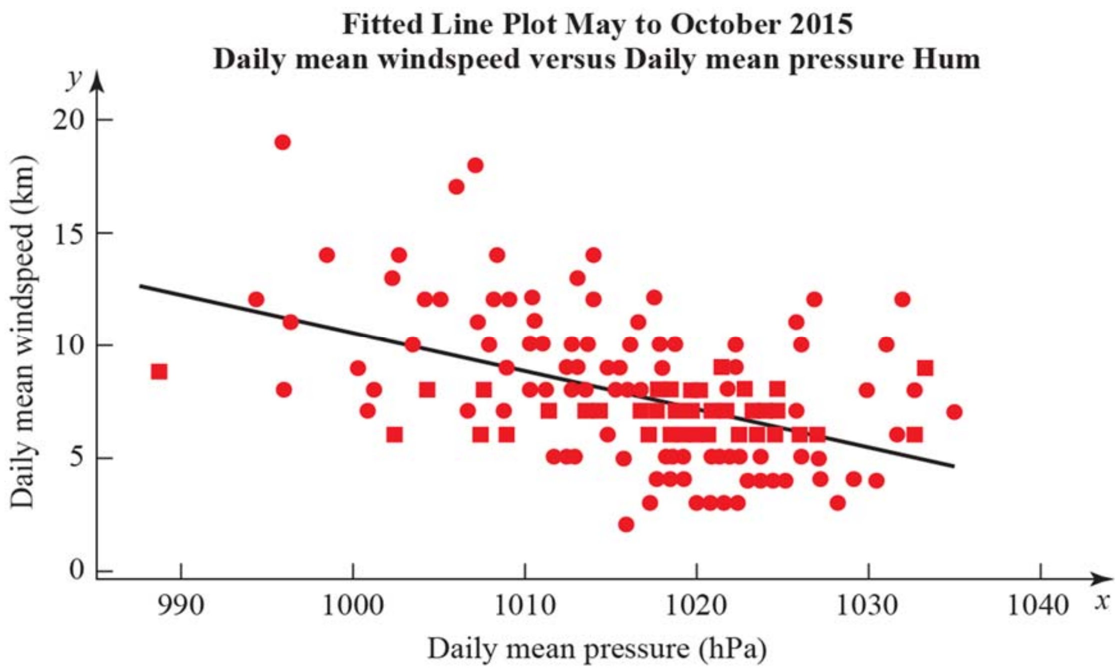

Figure 3

Correlation coefficient.
Daily mean winds and Daily mean pressure = −0.477 p-value < 0.001.
Regression summary output for daily mean wind speed versus daily mean pressure.

	Coefficients	Lower 95%	Upper 95%
Intercept	180.00	133.5424	226.4128
Daily Mean Pressure (hPa) Gradient	−0.1694	−0.21512	−0.12377

(a) State what is measured by the product moment correlation coefficient.

(1)

(b) Comment on the correlation between the two variables.

(1)

(c) Give an interpretation of the correlation between the two variables.

(1)

(d) Test at 5% significance level whether or not the product moment correlation coefficient for the population is less than zero. State your hypotheses clearly.

(3)

(e) Write down the regression model for daily mean wind speed versus daily mean pressure.

(2)

(f) Interpret the gradient of the line of regression stated in part **e**.

(1)

(g) The regression model (equation of regression) was used to predict the daily mean wind speed of 11.15 knots for a daily mean pressure of 995 hPa. Comment on the accuracy of this prediction.

(1)

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5. $$\mathrm{P}(E) = 0.25,\ \mathrm{P}(F) = 0.4 \text{ and } \mathrm{P}(E \cap F) = 0.12$$

(a) Find $\mathrm{P}(E'|F')$

(2)

(b) Explain, showing your working, whether or not E and F are statistically independent. Give reasons for your answer.

(2)

The event G has $\mathrm{P}(G) = 0.15$.

The events E and G are mutually exclusive and the events F and G are independent.

(c) Draw a Venn diagram to illustrate the events E, F and G, giving the probabilities for each region.

(5)

(d) Find $\mathrm{P}([F \cup G]')$

(2)

(Total 11 marks)

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6. In a town, 54% of the residents are female and 46% are male. A random sample of 200 residents is chosen from the town. Using a suitable approximation, find the probability that more than half the sample are female.

(Total 6 marks)

SECTION B: MECHANICS

Answer ALL questions.

7. Two identical 5 m light see-saws are joined at their ends. Robert, who weighs 80 kg, stands on top of the joint. The distance between Robert and each of the pivots is 2 m. Poppy and Quentin stand on the two remaining ends of the see-saws, as shown in Figure 4. Poppy weighs p kg and Quentin weighs q kg. The system is in equilibrium.

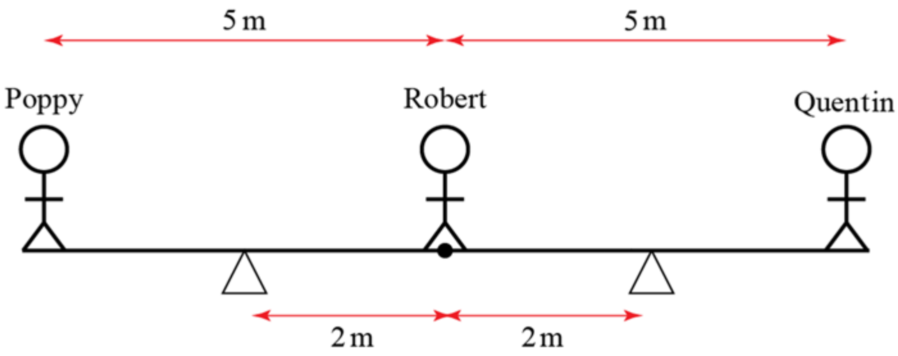

Figure 4

Show that, to the nearest whole number, $p + q = 53$.

(Total 8 marks)

8. Figure 5 shows an object of 3 kg sitting on a plane inclined at an angle θ to the horizontal. The coefficient of friction between the object and the plane is μ. The system is in limiting equilibrium.

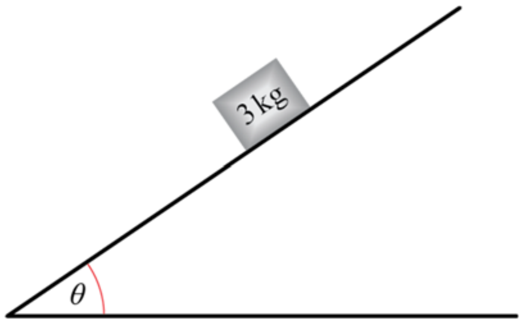

Figure 5

(a) Draw a diagram showing all the forces acting on the object. Describe the origin of each force using words.

(3)

(b) By resolving forces in two perpendicular directions, show that $\mu = \tan\theta$.

(6)

(c) Hence, determine whether or not the object slips if $\mu = 0.3$ and $\theta = 30°$.

(4)

(d) As θ approaches 90°, state whether an object of any mass could remain in equilibrium. Explain your answer.

(2)

(Total 15 marks)

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9. A projectile is launched at 8 m s^{-1} from the origin at a 60° angle to the horizontal. Find the length of time for which the particle is at least 2 m above its launch point.

(In this question, take $g = 9.8$ m s^{-2}.)

(Total 8 marks)

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10. A 0.5 kg particle experiences two forces, $\mathbf{A} = (2\mathbf{i} - \mathbf{j})$ N and $\mathbf{B} = \mathbf{I}$ N. Initially, the particle is at rest and has position vector $(3\mathbf{i} + 4\mathbf{j})$ m.

(a) Find the x and y coordinates of the particle t seconds later.

(9)

(b) Explain why the particle never returns to its starting point.

(2)

(c) Describe a physical situation which this mathematical model could represent and give physical meanings to **A** and **B**.

(2)

(Total 13 marks)

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11. The position, $\mathbf{r}$, of a planet orbiting a star at time t is given by $\mathbf{r} = \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$.

(a) Find the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the planet in terms of t.

(3)

(b) Show that $\mathbf{a} = -4\mathbf{r}$.

(1)

(c) Sketch the trajectory of the particle and draw arrows showing its velocity and acceleration when $t = 0$.

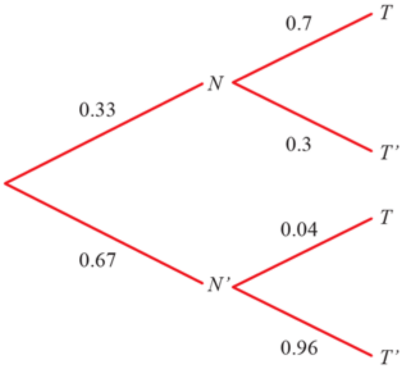
(2)

(Total 6 marks)

TOTAL FOR PAPER IS 100

G1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Linear association between e and f.	B1	1.2	2nd Know and understand the language of correlation and regression.
		(1)		
b	It requires extropolation and hence it may be unreliable.	B1	1.2	4th Understand the concepts of interpolation and extrapolation.
		(1)		
c	Fuel consumption (f)	B1	1.2	2nd Know and understand the language of correlation and regression.
		(1)		
d	A hypothesis test is a statistical test that is used to determine whether there is enough evidence in a sample of data to infer that a certain condition is true for the entire population.	B1	1.2	5th Understand the language of hypothesis testing.
		(1)		
e	$H_0 : \rho = 0$, $H_1 : \rho < 0$ Critical value = -0.3665 $-0.803 < -0.3665$ (test statistic in critical region) Reject H_0 There is evidence that the product moment correlation coeficient for CO_2 emissions and fuel consumption is less than zero.	B1 M1 A1	2.5 1.1b 2.2b	6th Carry out a hypothesis test for zero correlation.
		(3)		
				(7 marks)

Notes

G2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a		**B1**	2.5	3rd Draw and use tree diagrams with three branches and/or three levels.
	Let N~ new tyre and T~ tracking P(N) = 0.33 and P(T) = 0.67 0.7, 0.3, 0.04 and 0.96	**B1** **B1**	1.1b 1.1b	
		(3)		
b	P(exactly one defect) = $0.33 \times 0.3 + 0.67 \times 0.04$	**M1**	3.1b	5th Understand the language and notation of conditional probability.
	= 0.1258	**A1**	1.1b	
		(2)		
c	1 − P(no defects) =1− $0.67 \times 0.96 \times 0.65$	**M1**	3.1b	5th Understand the language and notation of conditional probability.
	= 0.5819… awrt 0.582 (3 d.p.)	**A1**	1.1b	
		(2)		
d	To have their cars checked regularly as there is over a 50 % chance they need new tyres, tracking or brake pads.	**B1**	3.2a	5th Understand the language and notation of conditional probability.
		(1)		
				(8 marks)

Notes

G3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Bell shaped.	**B1**	2.2a	5th Understand the basic features of the normal distribution including parameters, shape and notation.
		(1)		
b	X ~ Daily mean pressure $X \sim N(1006, 4.4^2)$	**M1**	3.3	5th Calculate probabilities for the standard normal distribution using a calculator.
	$P(X < 1000) = 0.0863$	**A1**	1.1b	
		(2)		
c	A sensible reason. For example, The tails of a Normal distribution are infinite. Cannot rule out extreme events.	**B1**	2.4	5th Understand the basic features of the normal distribution including parameters, shape and notation.
		(1)		

d	Comparison and sensible comment on means. For example, The mean daily mean pressure for Beijing is less than Jacksonville. This suggests better weather in Jacksonville. Comparison and sensible comment on standard deviations. For example, The standard deviation for Beijing is greater than that for Jacksonville. This suggests more consistent weather in Jacksonville. Student claim could be correct.	**B1** **B1** **B1** **B1**	2.2b 2.2b 2.2b 2.2b	8th Solve real-life problems in context using probability distributions.
		(4)		
				(8 marks)

Notes

a

Do not accept symmetrical with no discription of the shape.

d

B2 for Suggests better weather in Jacksonville but less consistent.

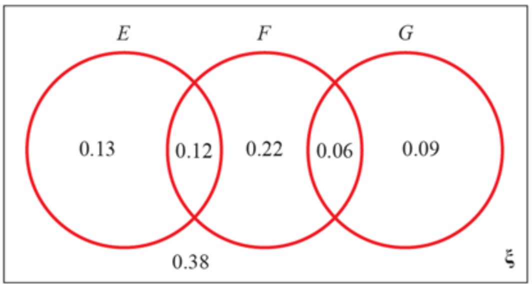
G4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Linear association between two variables.	B1	1.2	2nd Know and understand the language of correlation and regression.
		(1)		
b	Negative correlation.	B1	1.2	2nd Know and understand the language of correlation and regression.
		(1)		
c	As daily mean pressure increases (rises) daily mean wind speed decreases (falls) in Hurn May to October in 2015. or As daily mean pressure decreases (falls) daily mean wind speed increases (rises) in Hurn May to October in 2015.	B1	3.2	5th Interpret the PPMC as a measure of correlation.
		(1)		
d	$H_0 : \rho = 0$, $H_1 : \rho < 0$ p-value < 0.05 There is evidence to reject H_0. There is (strong) evidence of negative correlation between the daily mean wind speed and daily mean pressure.	B1 M1 A1	2.5 1.1b 2.2b	6th Carry out a hypothesis test for zero correlation.
		(3)		
e	Daily mean wind speed = 180 − 0.170 × daily mean pressure.	B2	1.1b	4th Use the principles of bivariate data analysis in the context of the large data set.
		(2)		

f	The regression model suggests for every hPa increase in daily mean pressure the daily mean wind speed decreases by 0.1694 knots. or The regression model suggests for every hPa decrease in daily mean pressure the daily mean wind speed increases by 0.1694 knots.	**B1**	3.2	4th Use the principles of bivariate data analysis in the context of the large data set.
		(1)		
g	Sensible comment. For example, Not very accurate as very few or no points Not very accurate as near the bottom range for the data.	**B1**	3.5b	4th Make predictions using the regression line within the range of the data.
		(1)		
				(10 marks)

Notes

e

B1 $y = 180.0 - 0.1694x$ unless x and y are defined.

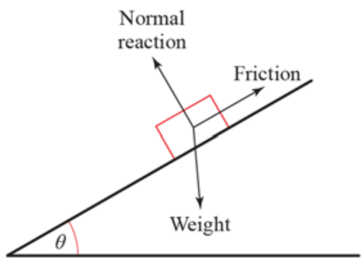
G5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\text{P}(E'\|F') = \dfrac{\text{P}(E' \cap F')}{\text{P}(F')}$ or $\dfrac{0.47}{0.6}$	M1	3.1a	4th Calculate probabilities using set notation.
	$= \dfrac{47}{60}$ or 0.783 (3 s.f.)	A1	1.1b	
		(2)		
B	$\text{P}(E) \times \text{P}(F) = 0.25 \times 0.4 = 0.1 \neq \text{P}(E \cap F) = 0.12$	M1	2.1	4th Understand and use the definition of independence in probability calculations.
	So, E and F are not statistically independent.	A1	2.4	
		(2)		
c	 Use of independence and all values in G correct. All values correct.	B1 M1A1 M1A1	2.5 3.1a 1.1b 1.1b 1.1b	3rd Understand and use Venn diagrams for multiple events.
		(5)		
d	$\text{P}([F \cup G]') = 0.13 + 0.38$	M1	3.1a	4th Calculate probabilities using set notation.
	$= 0.51$	A1	1.1b	
		(2)		
				(11 marks)

Notes

G6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	$X \sim \mathrm{B}(200, 0.54)$	**B1**	3.3	7th
	$Y \sim \mathrm{N}(108, 49.68)$	**B2**	3.1b	Use the normal distribution to approximate a binomial distribution.
	$\mathrm{P}(X > 100) = \mathrm{P}(X \geqslant 101)$	**M1**	3.4	
	$= \mathrm{P}\left(Z \ldots \dfrac{100.5 - 108}{\sqrt{49.68}}\right)$	**M1**	1.1b	
	$= \mathrm{P}(Z \geqslant -1.06...) = 0.8554$	**A1**	1.1b	
				(6 marks)

Notes

G7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Moment on see-saw is force × distance from pivot.	**M1**	1.1a	5th Solve equilibrium problems involving horizontal bars.
	Moment on Poppy's see-saw due to Poppy is $pg \times 3 = 3pg$ (N m)	**M1**	2.2a	
	Force on Bob due to Poppy is $\frac{3pg}{2}$ (N)	**A1**	2.2a	
	Force on Bob due to Quentin is $\frac{3qg}{2}$ (N)	**A1**	2.2a	
	Total force on Bob is $\frac{3}{2}(p+q)g$ (N)	**M1**	2.2a	
	Weight of Bob is $80g$ (N)	**M1**	1.1b	
	Forces are equal so $\frac{3}{2}(p+q)g = 80g$	**M1**	3.1b	
	$p + q = 53$ to the nearest whole number.	**A1**	2.4	
				(8 marks)

G8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	 B1 for each correct force with correct label.	**B3**	2.5	3rd Draw force diagrams.
		(3)		
b	Resolve horizontally/vertically or along/perp to plane.	**M1**	1.1b	7th The concept of limiting equilibrium.
	$R = 3g\cos\theta$	**A1**	1.1b	
	$F = 3g\sin\theta$	**A1**	1.1b	
	Limiting equilibrium means $\mu R = F$ $\mu R = 3\mu g\cos\theta$	**A1**	1.1b	
	$3\mu g\cos\theta = 3g\sin\theta$	**M1**	1.1b	
	$\mu = \tan\theta$	**A1**	1.1b	
		(6)		
c	$\tan 30 = 0.577\ldots$	**A1**	3.1a	7th The concept of limiting equilibrium.
	For limiting equilibrium, $\mu = 0.577\ldots$	**M1**	3.1a	
	But $\mu = 0.3$ so less friction.	**M1**	3.1a	
	Hence the object slips.	**A1**	3.2a	
		(4)		
d	No object would remain in equilibrium, because normal reaction becomes zero.	**B1** **A1**	3.2a	7th The concept of limiting equilibrium.
		(2)		
				(15 marks)

G9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Suvat equation.	**M1**	3.1a	8th Derive formulae for projectile motion.
	$y = 8t\sin 60 - \frac{1}{2}gt^2$	**M1**	1.1b	
	$= 4\sqrt{3}t - 4.9t^2$ (allow awrt 6.9)	**A1**	1.1b	
	Solve $y = 2$	**M1**	1.1a	
	$t = 0.404\ldots$ or $t = 1.009\ldots$ (accept awrt 0.40 and 1.01)	**A2**	1.1b	
	Time spent above 2 m is difference.	**M1**	2.4	
	0.605… (s) (accept awrt 0.61)	**A1ft**	3.4a	
				(8 marks)

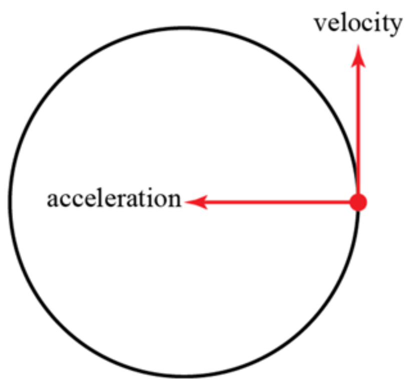
Notes

G10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Resultant force is **A** + **B**	M1	3.1b	5th Use Newton's second law to model motion in two directions.
	$= 3\mathbf{i} - \mathbf{j}$ (N)	A1	1.1b	
	Use of Newton's 2nd Law.	M1	3.1b	
	$\mathbf{a} = \dfrac{F}{m}$	M1	1.1b	
	$6\mathbf{i} - 2\mathbf{j}$ (m s^{-2})	A1	1.1b	
	$\mathbf{s} = \mathbf{s}_0 + \dfrac{1}{2}\mathbf{a}t^2$	M1	1.1a	
	$= 3\mathbf{i} + 4\mathbf{j} + \dfrac{1}{2}(6\mathbf{i} - 2\mathbf{j})t^2$	M1	1.1b	
	$x = 3 + 3t^2$	A1	1.1b	
	$y = 4 - t^2$	A1	1.1b	
		(9)		
b	$x = 3 + 3t^2 > 0$ for all $t > 0$	M1	2.4	4th Complete proofs by deduction and direct algebraic methods.
	so $x \neq 3$	A1	2.2a	
		(2)		
c	Anything resonable. For example, a ball in a river with wind. Descriptions of **A** and **B**. For example, **A** is force due to water. For example, **B** is force due to wind.	B1 B1	3.5 3.5	3rd Understand assumptions common in mathematical modelling.
		(2)		
				(13 marks)

Notes

b

Accept any valid argument (For example, equivalent argument for y)

G11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Differentiate **r** w.r.t. time	**M1**	1.1a	8th Solve general kinematics problems using calculus of vectors.
	$\mathbf{v} = \begin{pmatrix} -2\sin 2t \\ 2\cos 2t \end{pmatrix}$	**A1**	1.1b	
	$\mathbf{a} = \begin{pmatrix} -4\cos 2t \\ -4\sin 2t \end{pmatrix}$	**A1**	1.1b	
		(3)		
b	$\mathbf{a} = -4\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} = -4\mathbf{r}$	**B1**	2.2a	8th Solve general kinematics problems in a range of contexts using vectors.
		(1)		
c	 Diagram of circular orbit with velocity tangent to circle and acceleration pointing towards centre. Velocity must be in vertical direction.	**B1** **B1**	2.5 2.5	8th Solve general kinematics problems in a range of contexts using vectors.
		(2)		
				(6 marks)

Notes

c

B1 for correct velocity direction

B1 for correct acceleration direction

Pearson Edexcel Level 3
GCE Mathematics ## Advanced Level ## Paper 3: Statistics & Mechanics

Practice Set 3 **Time: 2 hours**	**Paper Reference(s)** **9MA0/03**

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SECTION A: STATISTICS

Answer ALL questions.

1. The distributions for the heights for a sample of females and males at a UK university can be modelled using normal distributions with mean 165 cm, standard deviation 9 cm and mean 178 cm, standard deviation 10 cm respectively.

A female's height of 177 cm and a male's height of 190 cm are both 12 cm above their means.

By calculating z-values, or otherwise, explain which is relatively taller.

(Total 4 marks)

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2. The table shows some data collected on the temperature, in °C, of a cup of coffee, c, and the time, t in minutes, after which it was made.

t	0	2	4	5	7	11	13	17	25
c	81.9	75.9	70.1	65.1	60.9	51.9	50.8	45.1	39.2

The data is coded using the changes of variable $x = t$ and $y = \log_{10} c$.

The regression line of y on x is found to be $y = 1.89 - 0.0131x$.

(a) Given that the data can be modelled by an equation of the form $c = ab^t$ where a and b are constants, find the values of a and b.

(3)

(b) Give an interpretation of the constant b in this equation.

(1)

(c) Explain why this model is not reliable for estimating the temperature of the coffee after an hour.

(1)

(Total 5 marks)

---

3. The table below shows the number of gold, silver and bronze medals won by two teams in an athletics competition.

	Gold	**Silver**	**Bronze**
Team *A*	29	17	18
Team *C*	21	23	17

The events G, S and B are that a medal is gold, silver or bronze respectively. Let A be the event that team A won a medal and C team C won a medal. A medal winner is selected at random. Find

(a) P(G),

(2)

(b) P($[A \cap S]'$).

(2)

(c) Explain, showing your working, whether or not events S and A are statistically independent. Give reasons for your answer.

(2)

(d) Determine whether or not events B and C are mutually exclusive. Give a reason for your answer.

(2)

(e) Given that 30% of the gold medal winners are female, 60% of the silver medal winners are female and 40% of the bronze medal winners are female, find the probability that a randomly selected medal winner is female.

(2)

(Total 10 marks)

---

4. A certain type of cabbage has a mass M which is normally distributed with mean 900 g and standard deviation 100 g.

(a) Find P(M < 850).

(1)

10% of the cabbages are too light and 10% are too heavy to be packaged and sold at a fixed price.

(b) Find the minimum and maximum weights of the cabbages that are packaged.

(3)

(Total 4 marks)

---

5. The data and scatter diagram (Figure 1) show the weight of chickens, x kilograms, and the average weight, y grams, of eggs laid by a random sample of 10 chickens.

Weight of chickens (kg)	2.9	1.9	1.6	2.7	3.1	2.2	2.7	1.9	1.7	2.6
Average weight of eggs (g)	58	56	55	66	47	63	49	56	53	53

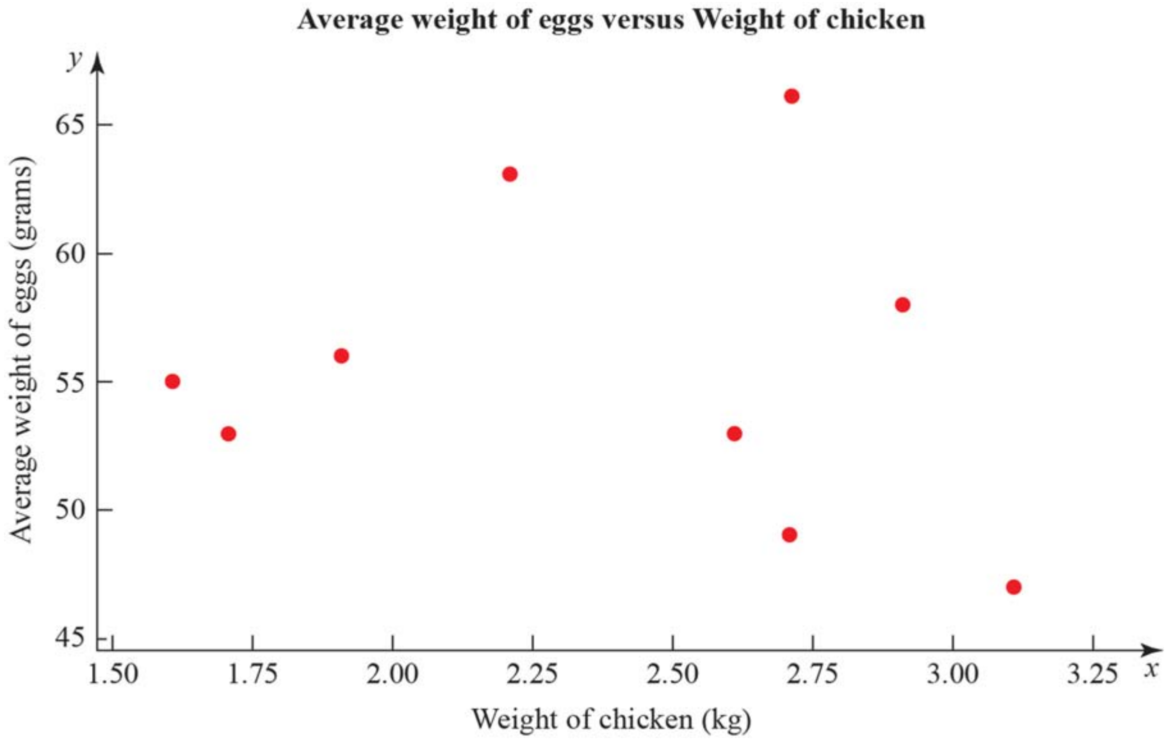


Figure 1

The product moment correlation coefficient for the average weight of eggs and weight of chickens is −0.136.

(a) Test for evidence of a negative population product moment correlation coefficient at the 2.5% significance level. Interpret this result in context.

(3)

(b) Explain why even if the population product moment correlation coefficient between two variables is close to zero there may still be a relationship between them.

(2)

(Total 5 marks)

6. (a) State the conditions under which the normal distribution may be used as an appoximation to the binomial distribution $X \sim B(n, p)$.

(2)

(b) Write down the mean and variance of the normal approximation to X in terms of n and p.

(2)

A manufacturer claims that more than 55% of its batteries last for at least 15 hours of continuous use.

(c) Write down a reason why the manufacturer should not justify their claim by testing all the batteries they produce.

(1)

To test the manufacturer's claim, a random sample of 300 batteries were tested.

(d) State the hypotheses for a one-tailed test of the manufacturer's claim.

(1)

(e) Given that 184 of the 300 batteries lasted for at least 15 hours of continuous use a normal approximation to test, at the 5% level of significance, whether or not the manufacturer's claim is justified.

(7)

(Total 13 marks)

---

7. The mean body temperature for women is normally distributed with mean 36.73 °C with variance 0.1482 $(°\text{C})^2$. Kay has a temperature of 38.1 °C.

(a) Calculate the probability of a woman having a temperature greater than 38.1 °C.

(2)

(b) Advise whether should Kay get medical advice. Give a reason for your advice.

(1)

(Total 3 marks)

---

8. To investigate if there is a correlation between daily mean temperature (°C) and daily mean pressure (hPa) the location Hurn 2015 was randomly selected from:

Camborne 2015	Camborne 1987
Hurn 2015	Hurn 1987
Leuchars 2015	Leuchars 1987
Leeming 2015	Leeming 1987
Heathrow 2015	Heathrow 1987

(Source: Pearson Edexcel GCE AS and A Level Mathematics data set.)

(a) State the definition of a test statistic.

(1)

(b) The product moment correlation coefficient between daily mean temperature and daily mean pressure for these data is −0.258 with a p-value of 0.001. Use a 5% significance level to test whether or not there is evidence of a correlation between the daily mean temperature and daily mean pressure.

(3)

(c) The scatter diagram in Figure 2 shows daily mean temperature versus daily mean pressure, by season, for Hurn 2015. Give two interpretations on the split of the data between summer and autumn.

(2)

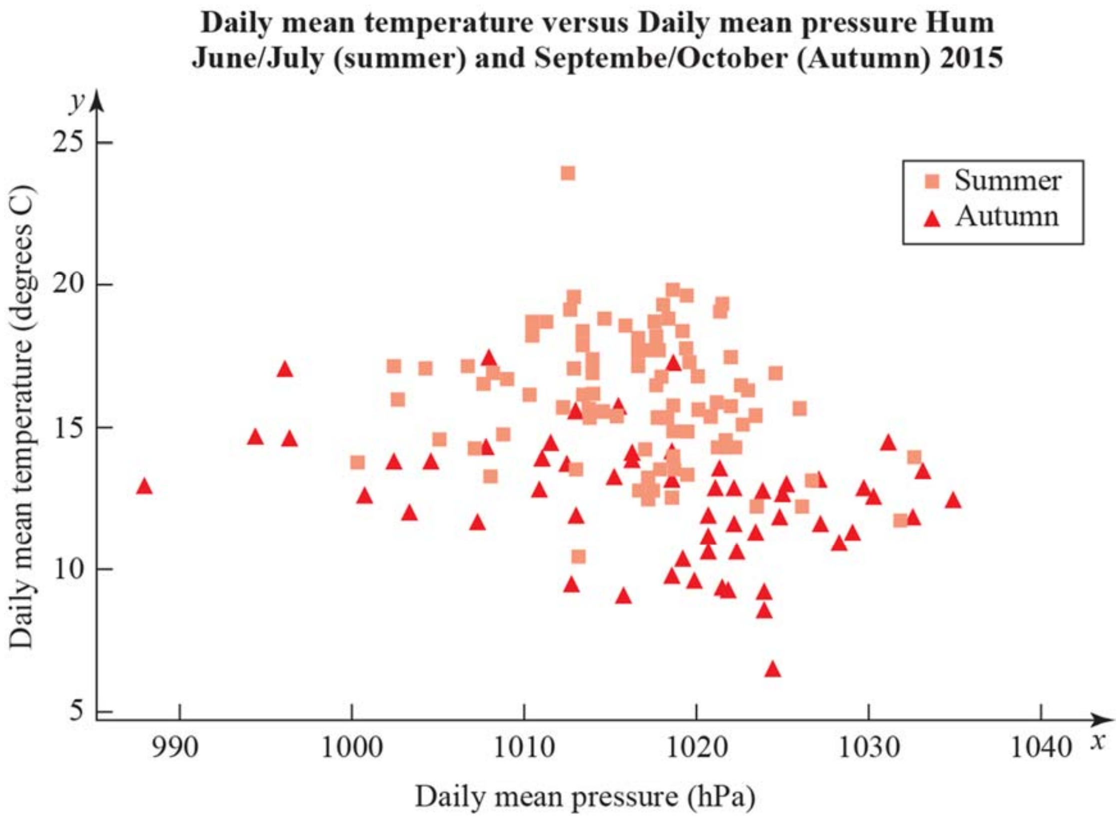


Figure 2

(Total 6 marks)

SECTION B: MECHANICS

Answer ALL questions.

9. At time t seconds, a 2 kg particle experiences a force **F** N, where $\mathbf{F} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} t + \begin{pmatrix} 6 \\ -12 \end{pmatrix} t^2$

(a) Find the acceleration of the particle at time t seconds.

(3)

The particle is initially at rest at the origin.

(b) Find the position of the particle at time t seconds.

(6)

(c) Find the particle's velocity when $t = 1$.

(3)

(Total 12 marks)

---

10. An archer shoots an arrow at 10 m s^{-1} from the origin and hits a target at $(10, -5)$ m. The initial velocity of the arrow is at an angle θ above the horizontal. The arrow is modelled as a particle moving freely under gravity.

(In this question, take $g = 10\text{ m s}^{-2}$.)

(a) Show that $(\tan\theta - 1)^2 = 1$.

(11)

(b) Find the possible values of θ.

(3)

(Total 14 marks)

---

11. Figure 3 shows a 5000 kg bus hanging 12 m over the edge of a cliff with 1000 kg of gold at the front. The gold sits on a wheeled cart. A group of n people, each weighing 70 kg, stands at the other end. The bus is 20 m long.

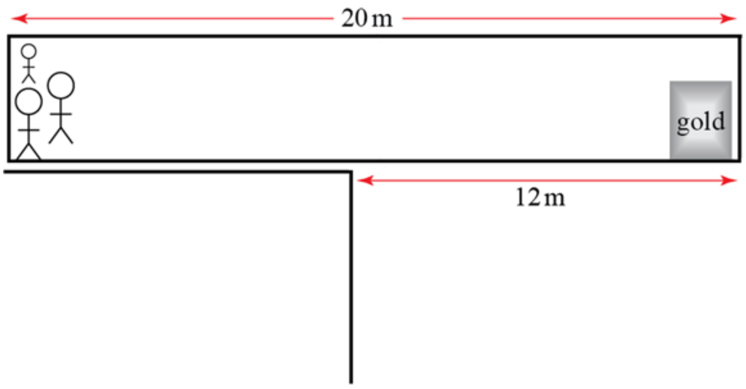


Figure 3

(a) Write down the total clockwise moment about the cliff edge in terms of n.

(7)

(b) Find the smallest number of people needed to stop the bus falling over the cliff.

(2)

(c) One person needs to walk to the end of the bus to retrieve the gold. Find the smallest number of people needed to stop the bus falling over the cliff in this situation, including the one retrieving the gold.

(4)

(Total 13 marks)

---

12. A car travels along a long, straight road for one hour, starting from rest. After t hours, its acceleration is a km h^{-2}, where $a = 180 - 360t$.

(a) Find the speed of the car, in km h^{-1} in terms of t.

(2)

The speed limit is 40 km h^{-1}.

(b) Find the range of times during which the car is breaking the speed limit. Give your answer in minutes.

(4)

(c) Find the average speed of the car over the whole journey.

(5)

(Total 11 marks)

TOTAL FOR PAPER IS 100

H1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	$X \sim$ females $X \sim \text{N}(165, 9^2)$, $Y \sim$ males $Y \sim \text{N}(178, 10^2)$	**M1**	3.3	5th Calculate probabilities for the standard normal distribution using a calculator.
	$\text{P}(X > 177) = \text{P}(Z > 1.33)$ (or = 0.0912)	**M1**	1.1b	
	$\text{P}(Y > 190) = \text{P}(Z > 1.20)$ (or = 0.1151)	**A1**	1.1b	
	Therefore the females are relatively taller.	**A1**	2.2a	
				(4 marks)

H2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\log_{10} c = 1.89 - 0.0131t$ $c = 10^{1.89-0.0131t}$ $c = 77.6 \times 0.970^t$ (3 s.f.)	**M1** **M1** **A1**	1.1a 1.1b 1.1b	6th Understand exponential models in bivariate data.
		(3)		
b	b is the proportional rate at which the temperature changes per minute.	**A1**	3.2a	6th Understand exponential models in bivariate data.
		(1)		
c	Extrapolation/out of the range of the data.	**A1**	2.4	4th Understand the concepts of interpolation and extrapolation.
		(1)		
				(5 marks)
	Notes			

H3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\frac{29+21}{29+21+17+23+18+17}=\frac{50}{125}$	M1	1.1b	2nd Calculate probabilities from relative frequency tables and real data.
	$= 0.4$	A1	1.1b	
		(2)		
b	$\frac{125-17}{125}=\frac{108}{125}$	M1	3.1a	4th Understand set notation.
	$= 0.864$	A1	1.1b	
		(2)		
c	$\mathrm{P}(S\cap A)=\frac{17}{125}=0.136\neq \mathrm{P}(S)\times \mathrm{P}(A)=\frac{40}{125}\times\frac{64}{125}=0.163\ldots$	M1	2.1	4th Understand and use the definition of independence in probability calculations.
	So, S and A are not statistically independent.	A1	2.4	
		(2)		
d	B and C are not mutally exclusive	B1	2.2a	3rd Understand and use the definition of mutually exclusive in probability calculations.
	Being in team C does not exclude the possibility of winning a bronze medal	B1	2.4	
		(2)		
e	$\frac{15+24+14}{125}=\frac{53}{125}$	M1	3.1b	5th Calculate conditional probabilities using formulae.
	$= 0.424$	A1	1.1b	
		(2)		
		(10 marks)		
	Notes			

H4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$P(M < 850) = 0.3085$ (using calculator)	**B1**	1.1b	5th Calculate probabilities for the standard normal distribution using a calculator.
		(1)		
b	$P(M < a) = 0.1$ and $P(M < b) = 0.9$	**M1**	3.1b	5th Calculate probabilities for the standard normal distribution using a calculator.
	(using calculator) $a = 772$ g	**A1**	1.1b	
	$b = 1028$ g	**A1**	1.1b	
		(3)		
				(4 marks)

Notes

H5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$H_0 : \rho = 0$, $H_1 : \rho < 0$	B1	2.5	6th
	Critical value = −0.6319	M1	1.1a	Carry out a hypothesis test for zero correlation.
	−0.6319 < −0.136 no evidence to reject H_0 (test statistic not in critical region)			
	There is insufficient evidence to suggest that the weight of chickens and average weight of eggs are negatively correlated.	A1	2.2b	
		(3)		
b	Sensible explanation. For example, correlation shows there is no (or extremely weak) linear realtionship between the two variables.	B1	1.2	7th Interpret the results of a hypothesis test for zero correlation.
	For example, there could be a non-linear relationship between the two variables.	B1	3.5b	
		(2)		
				(5 marks)

Notes

H6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	n is large	B1	1.2	5th Understand the binomial distribution (and its notation) and its use as a model.
	p is close to 0.5	B1	1.2	
		(2)		
b	Mean = np	B1	1.2	5th Understand the binomial distribution (and its notation) and its use as a model.
	Variance = $np(1-p)$	B1	1.2	
		(2)		
c	There would be no batteries left.	B1	2.4	5th Select and critique a sampling technique in a given context.
		(1)		
d	H_0: $p = 0.55$ H_1: $p > 0.55$	B1	2.5	5th Carry out 1-tail tests for the binomial distribution.
		(1)		
e	$X \sim N(165, 74.25)$ $P(X \geqslant 183.5)$ $= P\left(Z \ldots \dfrac{183.5-165}{\sqrt{74.25}}\right)$ $= P(Z \geqslant 2.146...)$ $=1 - 0.9838$ $= 0.0159$ Reject H_0, it is in the critical region. There is evidence to support the manufacturer's claim.	B1 M1 M1 A1 A1 M1 A1	3.3 3.4 1.1b 1.1b 1.1b 1.1b 2.2b	7th Interpret the results of a hypothesis test for the mean of a normal distribution.
		(7)		
				(13 marks)

Notes

H7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$X \sim$ women's body temperature $X \sim \text{N}(36.73, 0.1482)$	M1	3.3	5th
	$\text{P}(X > 38.1) = 0.000186$	B1	1.1b	Calculate probabilities for the standard normal distribution using a calculator.
		(2)		
b	Sensible reason. For example, Call the doctor as very unlikely the temperature would be so high.	B1	2.2a	8th Solve real-life problems in context using probability distributions.
		(1)		
				(3 marks)

Notes

H8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	A statistic that is calculated from sample data in order to test a hypothesis about a population.	**B1**	1.2	5th Understand the language of hypothesis testing.
		(1)		
b	$H_0 : \rho = 0$, $H_1 : \rho \neq 0$ p-value < 0.05 There is evidence to reject H_0 There is evidence (at 5% level) of a correlation between the daily mean temperature and daily mean pressure.	**B1** **M1** **A1**	2.5 1.1b 2.2b	6th Carry out a hypothesis test for zero correlation.
		(3)		
c	Two sensible interpretations or observations. For example, Two distinct distributions Similar gradients of regression line. Similar correlations for each season. Lower temperaure in autumn. More spread for the daily mean pressure in autumn.	**B2**	3.2a	4th Use the principles of bivariate data analysis in the context of the large data set.
		(2)		
				(6 marks)

Notes

H9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Use of Newton's second law.	**M1**	3.1b	8th Understand general kinematics problems with vectors.
	$\mathbf{a} = \frac{\mathbf{F}}{2}$	**M1**	1.1b	
	$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} t + \begin{pmatrix} 3 \\ -6 \end{pmatrix} t^2 \text{ (m s}^{-2}\text{)}$	**A1**	1.1b	
		(3)		
b	Integrate **a**	**M1**	1.1a	8th Solve general kinematics problems using calculus of vectors.
	$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} 1 \\ -2 \end{pmatrix} t^3 + \mathbf{c} \text{ (m s}^{-1}\text{)}$	**A1**	1.1b	
	$\mathbf{c} = 0$ because initially at rest.	**A1**	2.4	
	Integrate **v**	**M1**	1.1a	
	$\mathbf{r} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} t^3 + \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{2} \end{pmatrix} t^4 + \mathbf{c} \text{ (m)}$	**A1**	1.1b	
	$\mathbf{c} = 0$ because initially at origin.	**A1**	2.4	
		(6)		
c	Subsititute $t = 1$	**M1**	1.1a	6th Understand general kinematics problems with vectors.
	$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$	**M1**	1.1b	
	$= \begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{(m s}^{-1}\text{)}$	**A1**	1.1b	
		(3)		
				(12 marks)

Notes

H11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Moment from bus = $5000 \times 2 \times g$	M1	3.1a	5th Find resultant moments by considering direction.
	$= 10\,000g$ (N m)	A1	1.1b	
	Moment from gold = $1000 \times 12 \times g$	M1	3.1b	
	$= 12\,000g$ (N m)	A1	1.1b	
	Moment from people = $70 \times 8 \times n \times g$	M1	3.1a	
	$= 560ng$ (N m)	A1	1.1b	
	Total moment = $(22\,000 - 560n)g$ (N m)	A1	1.1b	
		(7)		
b	Forming an equation or inequality for n and solving to find ($n = 39.28\ldots$)	M1	1.1b	5th Solve equilibrium problems involving horizontal bars.
	Need 40 people.	A1	3.2a	
		(2)		
c	New moment from gold and extra person is $1070 \times 12 \times g$ (N)	M1	3.1a	5th Solve equilibrium problems involving horizontal bars.
	New total moment = $(22840 - 560n)g$ (N m)	M1	1.1b	
	$n = 40.78\ldots$	A1	3.2a	
	42 people (including the extra)	A1	2.4	
		(4)		
				(13 marks)

H10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Use of suvat equations	M1	1.1a	8th Derive formulae for projectile motion.
	$x = 10t\cos\theta$	A1	1.1b	
	$y = 10t\sin\theta - \frac{1}{2}gt^2$	M1	1.1b	
	$= 10t\sin\theta - 5t^2$	A1	1.1b	
	Substitute $x = 10$ and $y = -5$	M1	1.1a	
	Solve x equation for t	M1	1.1b	
	$t = \frac{1}{\cos\theta}$	A1	1.1b	
	Substitute into y equation	M1	1.1a	
	$-5 = 10\tan\theta - 5\sec^2\theta$	A1	2.1	
	Use of $\sec^2\theta = 1 + \tan^2\theta$	M1	2.1	
	$(\tan\theta - 1)^2 = 1$ legitimately obtained	A1	2.1	
		(11)		
b	Solve for $\tan\theta$	M1	1.1a	8th Solve problems in unfamiliar contexts using the concepts of friction and motion.
	$\tan\theta = 0$ or $\tan\theta = 2$	A1	1.1b	
	$\theta = 0$ or 63.43…(°) (accept awrt 63)	A1	1.1b	
		(3)		
				(14 marks)

Notes

H12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Integrate a w.r.t. t	**M1**	1.1a	5th Use integration to determine functions for velocity and/or displacement.
	$a = 180t - 180t^2$	**A1**	1.1b	
		(2)		
b	$180t - 180t^2 > 40$	**M1**	3.1a	7th Solve general kinematics problems in less familiar contexts.
	$20(3t-2)(3t-1) < 0$	**A1**	1.1b	
	$\frac{1}{3} < t < \frac{2}{3}$	**A1**	2.4	
	Breaking the speed limit between 20 and 40 minutes.	**A1**	3.2a	
		(4)		
c	Integrate v w.r.t. t	**M1**	1.1a	5th Use integration to determine functions for velocity and/or displacement.
	$x = 90t^2 - 60t^3\ (+C)$	**A1**	1.1b	
	When $t = 1$, $x = 30$	**A1**	3.1b	
	Average speed $= \frac{\text{distance}}{\text{time}}$	**M1**	1.1b	
	$30\ \text{km h}^{-1}$	**A1**	1.1b	
		(5)		
				(11 marks)

Notes

Pearson Edexcel Level 3
GCE Mathematics ## Advanced Level ## Paper 3: Statistics & Mechanics

Practice Set 4 **Time: 2 hours**	**Paper Reference(s)** **9MA0/03**
You must have: **Mathematical Formulae and Statistical Tables, calculator**	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this paper. The total is 100.
- For each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

SECTION A: STATISTICS

Answer ALL questions.

1. The number of bacteria, n thousand per cm^3, in a sample of liquid is measured over a period of time, t, in hours. The data is shown in the table.

t	3.9	5.5	6.8	8.5	10.6	11.5	13.3	14.7	16.5	17.8
n	10.1	13.1	14.6	20.7	27.9	31.5	40	49.9	64.7	75.6

The data is coded using the changes of variable $x = t$ and $y = \log_{10} n$.

The regression line of y on x is found to be $y = 0.7606 + 0.0635x$.

(a) Given that the data can be modelled by an equation of the form $n = ab^t$ where a and b are constants, find the values of a and b.

(3)

(b) Give an interpretation of the constant a in this equation.

(1)

(c) Explain why this model is not reliable for estimating the number of bacteria after 24 hours.

(1)

(Total 5 marks)

2. In a factory, three machinists, Amy, Brad and Ceri, are used to sew shirts.

Amy sews 40% of the shirts.
Brad sews 35% of the shirts.
Ceri sews the rest of the shirts.

It is known that 5% of the shirts sewn by Amy are faulty, 2% of the shirts sewn by Brad are faulty and 3% of the shirts sewn by Ceri are faulty.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities.
(3)

A shirt is selected at random.

(b) Calculate the probability that the shirt is sewn by Brad and is not faulty.
(2)

(c) Calculate the probability that the shirt is faulty.
(2)

(d) Given that the shirt is faulty, find the probability that it was not sewn by Ceri.
(3)

(Total 10 marks)

---

3. The heights of a population of men are normally distributed with mean μ cm and standard deviation σ cm. It is known that 20% of the men are taller than 180 cm and 5% are shorter than 170 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.
(3)

(b) Find the value of μ and σ.
(7)

(c) Three men are selected at random, find the probability that they are all taller than 175 cm.
(2)

(Total 12 marks)

---

4. The data and scatter diagram in Figure 1 show the population, p, in millions, of a country taken t years since their first census.

t	0	10	20	30	40	50	60	70	80	90	100
p	238.4	252.1	251.3	279	318.7	361.1	439.2	548.2	683.3	846.4	1028.7

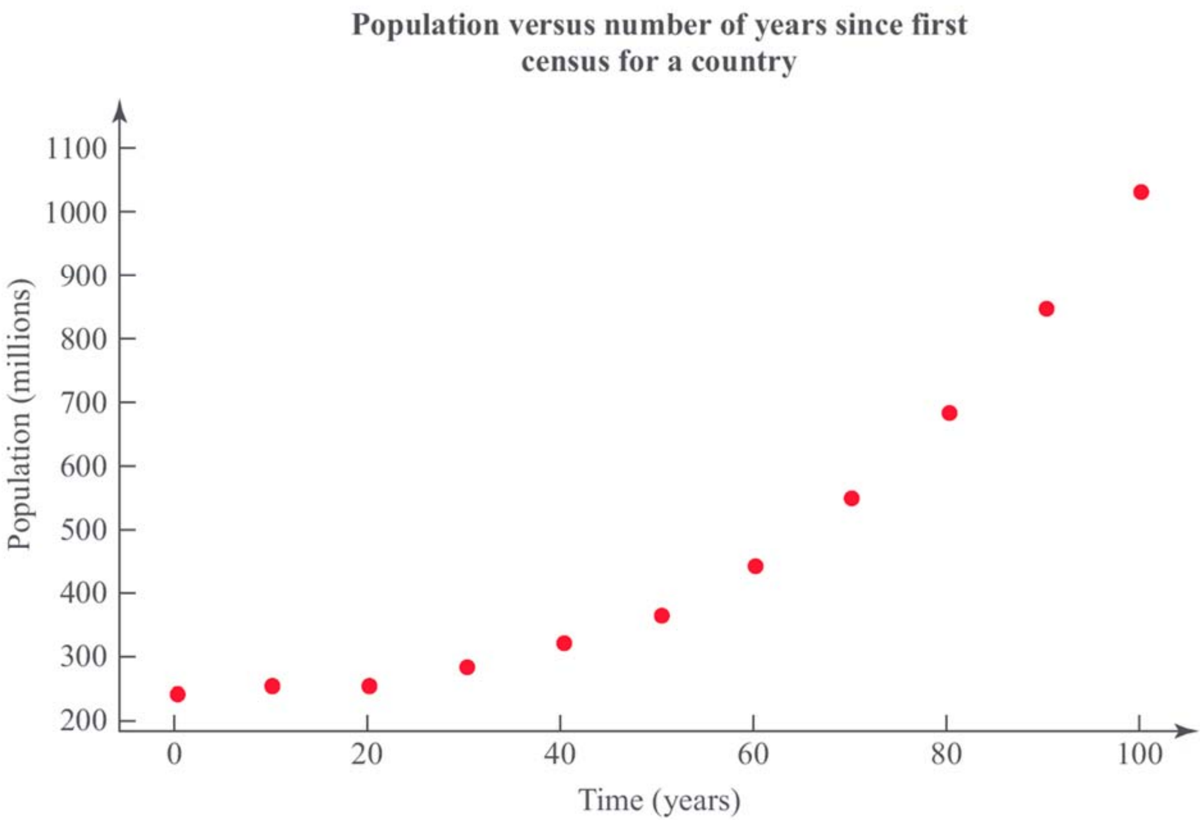


Figure 1

(a) Give a reason why the data is coded using the changes of variable $x = t$ and $y = \log_{10} p$.

(1)

(b) The product moment correlation coefficient for the coded data is $r = 0.9735$. Comment on r for this model.

(2)

(c) With reference to your answer to part (b), state whether a model in the form $p = ab^t$, where a and b are constants, is a good fit for this data.

(2)

(Total 5 marks)

5. A group of students were surveyed by a principal and $\frac{2}{3}$ were found to always hand in assignments on time. When questioned about their assignments $\frac{3}{5}$ said they always start their assignments on the day they are issued and, of those who always start their assignments on the day they are issued, $\frac{11}{20}$ hand them in on time.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly selected student

(i) always start their assignments on the day they are issued and hand them in on time,

(2)

(ii) does not always hand in assignments on time and does not start their assignments on the day they are issued.

(4)

(c) Determine whether or not always starting assignments on the day they are issued and handing them in on time are statistically independent. Give reasons for your answer.

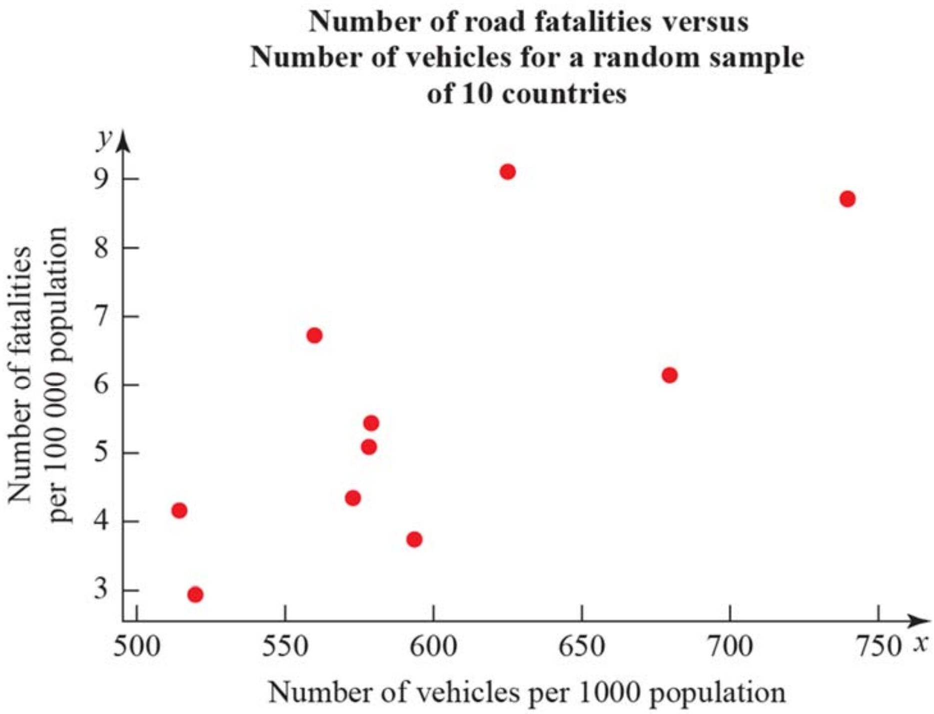
(2)

(Total 11 marks)

6. A researcher wishes to investigate if there is a positive correlation between the number of vehicles and the number of road fatalities in European countries.

He selects a random sample of 10 European countries and records the number of vehicles, v per 1000 people, and the number of road fatalities, r per 100 000 population, for a particular year. These are shown in the table and scatter diagrams in Figure 2.

Country	v	r
Austria	578	5.4
Belgium	559	6.7
France	578	5.1
Germany	572	4.3
Greece	624	9.1
Ireland	513	4.1
Italy	679	6.1
Luxembourg	739	8.7
Spain	593	3.7
UK	519	2.9



$r = -7.0 + 0.02y$

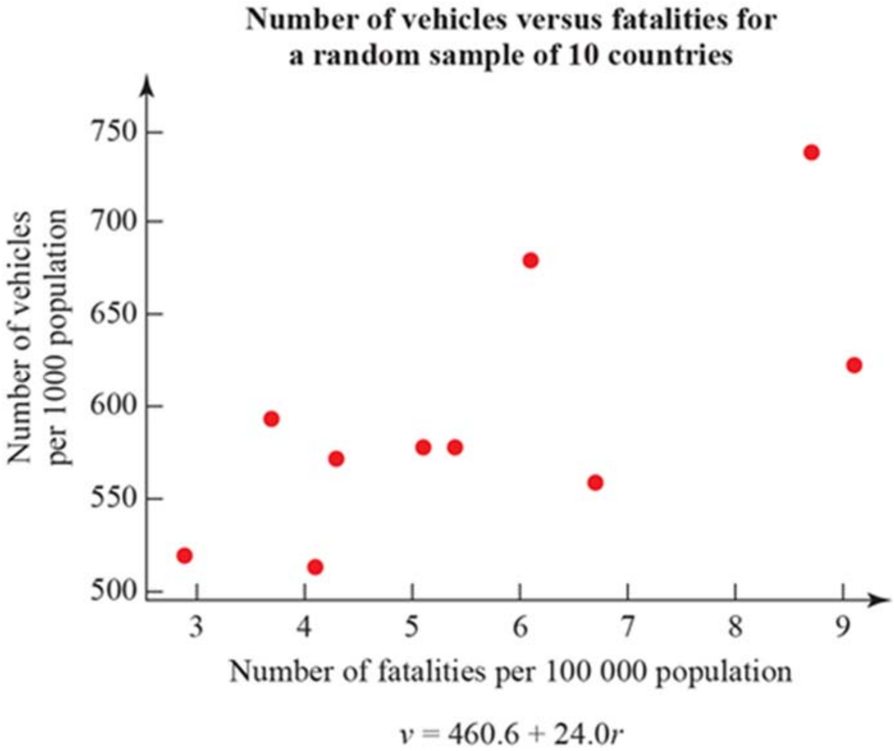


Figure 2

(a) What is the definition of a critical value?

(1)

(b) The product moment correlaton coefficient for v and r is 0.714. Use this value to test for positive correlation at the 5% significance level. Interpret your result in context.

(3)

(c) The researcher wishes to predict the number of road fatalities for a country with 650 vehicles per 1000 people. Write down the regression model he should use.

(1)

(d) State the dependent variable for the regression model in part (c).

(1)

(e) Monaco has 899 vehicles per 1000 people. Explain why the model stated in part (c) is not reliable for estimating the number of road fatalities in Monaco.

(1)

(Total 7 marks)

SECTION B: MECHANICS

Answer ALL questions.

7. An object has three different forces $\mathbf{F}_1$ N, $\mathbf{F}_2$ N and $\mathbf{F}_3$ N acting on its centre of mass.

$\mathbf{F}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{F}_2 = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$. The object is in equilibrium.

Find $\mathbf{F}_3$.

(Total 3 marks)

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8. A 0.1 kg inflatable ball floats on the surface of the sea. The current from the water underneath the ball exerts a force $\mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ N and the wind exerts a force of $\mathbf{W} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ N on the ball.

(a) Find the resultant force exerted on the ball.

(2)

(b) Calculate the acceleration of the ball.

(3)

Initially, the ball is at the origin and has velocity $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ N m s^{-1}.

(c) Find the x and y coordinates of the ball t s later.

(4)

(d) Find the distance travelled by the ball when $t = 10$ s.

(5)

(Total 14 marks)

---

9. Figure 3 shows Alice, who weighs 50 kg, sitting on the right-hand end of a light see-saw. Bob, who weighs 80 kg, stands on the opposite side at a distance x m from the end. The length of the see-saw is 4 m and it pivots about its centre.

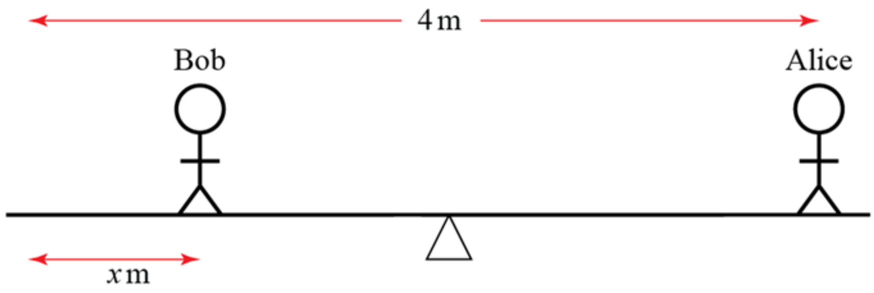

Figure 3

(a) Draw a diagram showing the forces acting on the see-saw due to the two people. Label the value of each force in newtons.

(2)

(b) Write down the total clockwise moment about the centre in terms of x.

(5)

(c) Find the value of x for which the see-saw is in equilibrium.

(2)

(d) Given that Bob remains on the opposite side to Alice, describe with inequalities the range of x for which the see-saw tilts towards Alice.

(2)

(e) Describe one limitation of this model.

(1)

(Total 12 marks)

10. A ball is launched from the origin with speed 1 m s^{-1}. Its velocity vector makes an angle θ above the horizontal. It travels over flat ground and is modelled as a particle moving freely under gravity, as shown in Figure 4.

(In this question, take $g = 10$ m s^{-2}.)

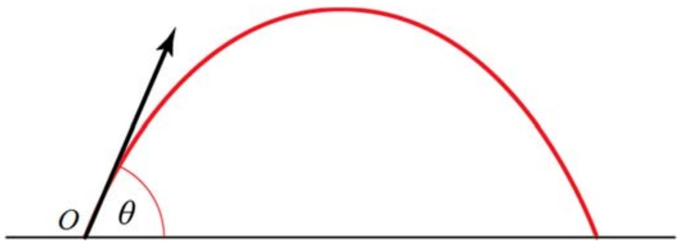

Figure 4

(a) Find the horizontal and vertical displacements of the particle at time t seconds. You should give your answer in terms of θ and t.

(4)

(b) Show that the horizontal distance travelled by the particle before it hits the ground is $\dfrac{\sin 2\theta}{10}$.

(5)

(c) Find the value θ for which the horizontal distance travelled is a maximum.

(2)

(d) Describe one limitation of this model.

(1)

(Total 12 marks)

11. Figure 5 shows a cylindrical object with mass 8 kg resting on two cylindrical bars of equal radius. The lines connecting the centre of each of the bars to the centre of the object make an angle of 40° to the vertical.

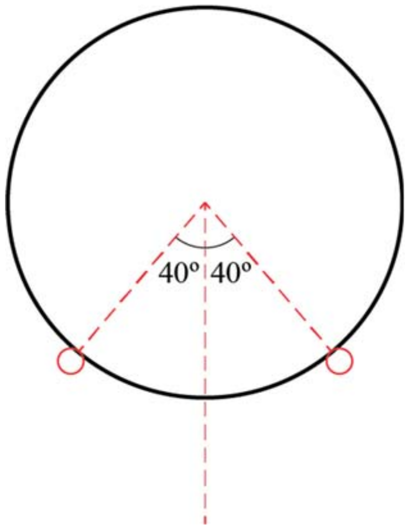

Figure 5

(a) Draw a diagram showing all the forces acting on the object. Describe each of the forces using words.

(2)

(b) Calculate the magnitude of the force on each of the bars due to the cylindrical object.

(7)

(Total 9 marks)

TOTAL FOR PAPER IS 100

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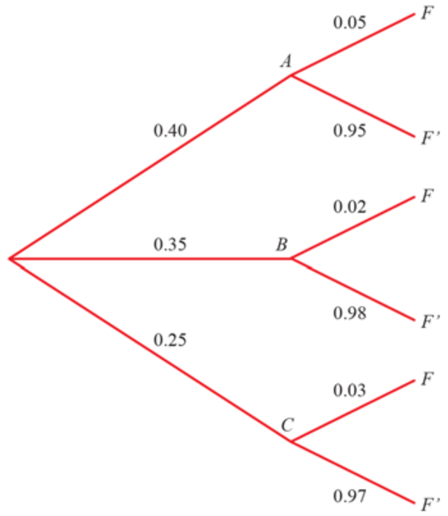
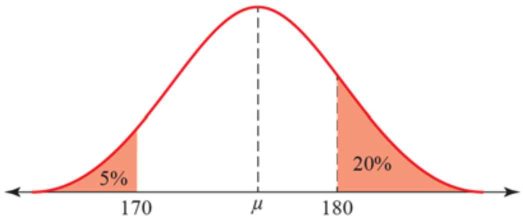
I1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\log n = 0.7606 + 0.0635t$ $c = 10^{0.7606+0.0635t}$ $c = 5.76 \times 1.16^t$ (3 s.f.)	M1 M1 A1	1.1a 1.1b 1.1b	6th Understand exponential models in bivariate data.
		(3)		
b	a is a constant of proportionality.	A1	3.2a	6th Understand exponential models in bivariate data.
		(1)		
c	Extrapolation/out of the range of the data.	A1	2.4	4th Understand the concepts of interpolation and extrapolation.
		(1)		
				(5 marks)
	Notes			

I2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	 Let F ~ faulty	**B1** **B1** **B1**	2.5 1.1b 1.1b	3rd Draw and use tree diagrams with three branches and/or three levels.
		(3)		
b	$\text{P}(B \cap F') = 0.35 \times 0.98$	**M1**	1.1b	5th Understand and calculate conditional probabilities in the context of tree diagrams.
	$= 0.343$	**A1**	1.1b	
		(2)		
c	$\text{P}(F) = 0.4 \times 0.05 + 0.35 \times 0.02 + 0.25 \times 0.03$	**M1**	1.1b	5th Understand and calculate conditional probabilities in the context of tree diagrams.
	$= 0.0345$	**A1**	1.1b	
		(2)		
d	$\text{P}(C'\|F) = \dfrac{\text{P}(C' \cap F)}{\text{P}(F)} = \dfrac{0.4 \times 0.05 + 0.35 \times 0.02}{0.0345} = \dfrac{0.027}{0.0345}$	**M1** **A1ft**	3.1b 1.2	5th Calculate conditional probabilities using formulae.
	$0.7826...$ or $\dfrac{18}{23}$ (accept awrt 0.783)	**A1**	1.1b	
		(3)		
				(10 marks)

Notes

I3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	 bell shaped	**B1**	1.2	5th Understand the basic features of the normal distribution including parameters, shape and notation.
	170, 180 on axis	**B1**	1.1b	
	5% and 20%	**B1**	1.1b	
		(3)		
b	P(X < 170) = 0.05 $\frac{170-\mu}{\sigma} = -1.6449$ $\mu = 170 + 1.6449\sigma$ P(X > 180) = 0.2 $\mu = 180 - 0.8416\sigma$ Solving simultaneously gives: μ = 176.615… (awrt 176.6) and σ = 4.021…(awrt 4.02)	**M1** **B1** **B1** **B1** **M1** **A1** **A1**	3.3 3.4 1.1b 3.4 1.1b 1.1b 1.1b	7th Find unknown means and/or standard deviations for normal distributions.
		(7)		
c	P(All three are taller than 175 cm) = $0.656\ldots^3$	**M1**	1.1b	5th Understand informally the link to probability distributions.
	= 0.282… (using calculator) awrt 0.282	**A1**	1.1b	
		(2)		
				(12 marks)

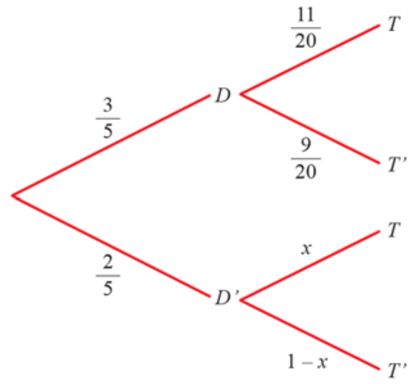
Notes

I4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	The data seems to follow an exponential distribution.	**B1**	2.4	6th Understand exponential models in bivariate data.
		(1)		
b	$r = 0.9735$ is close to 1	**B1**	2.2a	2nd Know and understand the language of correlation and regression.
	which gives a strong positive correlation.	**B1**	2.4	
		(2)		
c	Model is a good fit with a reason. For example, Very strong positive linear correlation between t and $\log_{10} p$. The **transformed data points** lie close (enough) to a straight line.	**B2**	3.2a	6th Understand exponential models in bivariate data.
		(2)		
				(5 marks)

Notes

c

B0 for just stating the model is a good fit with no reason.

I5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	 T = hand assignments in on time, D = start assignments on the day they are issued	**B1** **B1** **B1**	2.5 1.1b 1.1b	2nd Draw and use simple tree diagrams with two branches and two levels.
		(3)		
b i	$\mathrm{P}(T \cap D) = \mathrm{P}(T \mid D) \times \mathrm{P}(D)$	**M1**	3.1b	5th Understand and calculate conditional probabilities in the context of tree diagrams.
	$= \frac{3}{5} \times \frac{11}{20}$ $= \frac{33}{100}$ or 0.33	**A1**	1.1b	
		(2)		
b ii	$\frac{3}{5} \times \frac{11}{20} + x \times \frac{2}{5} = \frac{2}{3}$	**M1**	3.1b	5th Understand and calculate conditional probabilities in the context of tree diagrams.
	$x = \frac{101}{120}$ or 0.841…	**A1**	1.1b	
	$\mathrm{P}(T' \cap D') = = \frac{2}{5}\left(1 - \frac{101}{120}\right)$	**M1**	1.1b	
	$= \frac{19}{300}$ or 0.0633… (accept awrt 0.0633)	**A1**	1.1b	
		(4)		

c	$\text{P}(T \cap D) = \frac{33}{100} \neq \text{P}(T) \times \text{P}(D) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$	**M1**	2.1	4th Understand and use the definition of independence in probability calculations.
	So, T and D are not statistically independent.	**A1**	2.4	
		(2)		
				(11 marks)

Notes

b ii Alternative solution

$\text{P}(T' \cap D') = 1 - \text{P}(T \cup D)$

$\text{P}(T \cup D) = \frac{2}{3} + \frac{3}{5} - \frac{33}{100}$

$= \frac{281}{300}$

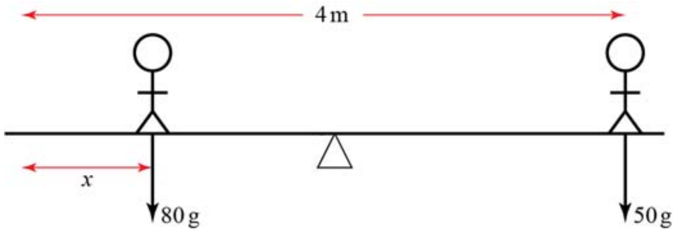
$\text{P}(T' \cap D') = 1 - \frac{281}{300} = \frac{19}{300}$

I6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	A critical value is the point (or points) on the scale of the test statistic beyond which we reject the null hypothesis.	**B1**	1.2	5th Understand the language of hypothesis testing.
		(1)		
b	$H_0 : \rho = 0$, $H_1 : \rho > 0$ Critical value = 0.5494 0.714 > 0.5494 (test statistic in critical region) There is evidence to reject H_0 There is evidence that there is a positive correlation between the number of vehicles and road traffic accidents.	**B1** **M1** **A1**	2.5 1.1b 2.2b	6th Carry out a hypothesis test for zero correlation.
		(3)		
c	$r = -7.0 + 0.02v$	**B1**	1.2	4th Make predictions using the regression line within the range of the data.
		(1)		
d	Road fatalities per 100 000 population.	**B1**	1.2	2nd Know and understand the language of correlation and regression.
		(1)		
e	Outside the range of the data used in the model. or This would require extrapolation.	**B1**	3.5b	4th Understand the concepts of interpolation and extrapolation.
		(1)		
				(7 marks)
	Notes			

I7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Moment from bus = $5000 \times 2 \times g$	**M1**	3.1a	5th Find resultant moments by considering direction.
	$= 10\,000g$ (N m)	**A1**	1.1b	
	Moment from gold = $1000 \times 12 \times g$	**M1**	3.1b	
	$= 12\,000g$ (N m)	**A1**	1.1b	
	Moment from people = $70 \times 8 \times n \times g$	**M1**	3.1a	
	$= 560ng$ (N m)	**A1**	1.1b	
	Total moment = $(22\,000 - 560n)g$ (N m)	**A1**	1.1b	
		(7)		
b	Forming an equation or inequality for n and solving to find ($n = 39.28\ldots$)	**M1**	1.1b	5th Solve equilibrium problems involving horizontal bars.
	Need 40 people.	**A1**	3.2a	
		(2)		
c	New moment from gold and extra person is $1070 \times 12 \times g$ (N)	**M1**	3.1a	5th Solve equilibrium problems involving horizontal bars.
	New total moment = $(22840 - 560n)g$ (N m)	**M1**	1.1b	
	$n = 40.78\ldots$	**A1**	3.2a	
	42 people (including the extra)	**A1**	2.4	
		(4)		
				(13 marks)

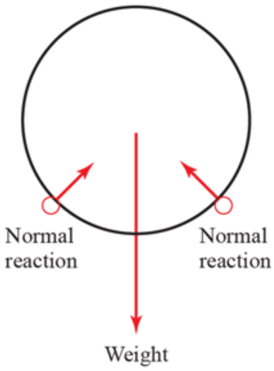
I8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Net force is **C** + **W**	M1	3.1b	4^{th} Calculate resultant forces using vectors.
	$=\begin{pmatrix}5\\-1\end{pmatrix}$	A1	1.1b	
		(2)		
b	Use of Newton's 2nd Law.	M1	3.1b	5th Use Newton's second law to model motion in two directions.
	$\mathbf{a}=\frac{F}{m}$	M1	1.1b	
	$=\begin{pmatrix}50\\-10\end{pmatrix}$	A1	1.1b	
		(3)		
c	$\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$	M1	1.1a	5th Use the equations of motion to solve problems in familiar contexts.
	$=\begin{pmatrix}1\\1\end{pmatrix}t+\frac{1}{2}\begin{pmatrix}50\\-10\end{pmatrix}t^2$	M1	1.1b	
	$x=t+25t^2$	A1	1.1b	
	$y=t-5t^2$	A1	1.1b	
		(4)		
d	Substitute $t=10$	M1	3.1b	5th Use the equations of motion to solve problems in familiar contexts.
	$x=2510$	A1	1.1b	
	$y=-490$	A1	1.1b	
	Distance travelled $=\sqrt{2510^2+(-490)^2}$	M1	1.1a	
	2557.38…(m) (Accept awrt 2560)	A1	3.2a	
		(5)		
		(14 marks)		

Notes

I9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	**Figure 1**  Force labels one mark each Allow explicit evaluation with g.	**B2**	2.5	4th Calculate moments.
		(2)		
b	Alice: Moment = $2 \times 50 \times g$	**M1**	1.1b	5th Calculate sums of moments.
	$= 100g$ (N m)	**A1**	1.1b	
	Bob: Moment = $(2 - x) \times 80 \times g$	**M1**	3.4	
	$= 80(2 - x)g$ (N m)	**A1**	1.1b	
	Total clockwise moment = $20g(4x - 3)$ (N m)	**A1**	1.1b	
		(5)		
c	Equating to 0 and solving	**M1**	3.4	5th Solve equilibrium problems involving horizontal bars.
	$x = 0.75$ (m)	**A1**	1.1b	
		(2)		
d	Identifying 2 as a limit	**M1**	2.4	7th Solve problems involving bodies on the point of tilting.
	So tilts towards Alice when $0.75 < x \leqslant 2$	**A1**	2.2a	
		(2)		
e	Any valid limitation. For example, Pivot not a point. Alice can't sit exactly on the end. The see-saw might bend.	**A1**	3.5	3rd Understand assumptions common in mathematical modelling.
		(1)		

I10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Use of $s = ut + \frac{1}{2}at^2$	M1	1.1a	6th Resolve velocity into horizontal and vertical components.
	Initial velocity is $(\cos\theta, \sin\theta)$	A1	3.4	
	$x = t\cos\theta$	A1	1.1b	
	$y = t\sin\theta - 5t^2$	B1	1.1b	
		(4)		
b	Solve $y = 0$ for t	M1	3.4	5th Model horizontal projection under gravity.
	$t(\sin\theta - 5t) = 0$	A1	1.1b	
	$t = 0$ or $t = \frac{\sin\theta}{5}$	A1	1.1b	
	$t = 0$ is initial position so $t = \frac{\sin\theta}{5}$	M1	2.4	
	$x = \frac{\cos\theta\sin\theta}{5} = \frac{2\sin\theta\cos\theta}{10} = \frac{\sin 2\theta}{10}$	A1	1.1b	
		(5)		
c	Sketch of $\sin 2\theta$ or other legitimate method.	M1	2.2a	6th
	Maximum is at $\theta = 45°$	A1	2.4	Resolve velocity into horizontal and vertical components.
		(2)		
d	Correct limitation. For example, air resistance.	B1	3.5b	3rd Understand assumptions common in mathematical modelling.
		(1)		
				(12 marks)

Notes

I11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a				3rd Draw force diagrams.
	One correct force with correct label.	**B1**	2.5	
	Two more correct forces with correct labels.	**B1**	2.5	
		(2)		
b	Resolve vertically.	**M1**	1.1b	5th Calculate resultant forces in perpendicular directions.
	Weight = $8g$	**M1**	1.1b	
	= 78.4	**M1**	1.1b	
	Vertical part of normal reaction is $2R\cos 40$	**A1**	1.1b	
	$2R\cos 40 = 78.4$	**M1**	1.1b	
	Solve for R	**M1**	1.1b	
	$R = 51.171\ldots$ (N) accept awrt 51	**A1**	1.1b	
		(7)		
				(9 marks)