

## Monday 19 October 2020

## Afternoon

## Mathematics

Advanced
Paper 32: Mechanics

You must have:<br>Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

## Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.


## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50 . There are 5 questions.
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 A rough plane is inclined to the horizontal at an angle $\alpha$, where $\tan \alpha=\frac{3}{4}$
A brick $P$ of mass $m$ is placed on the plane.
The coefficient of friction between $P$ and the plane is $\mu$
Brick $P$ is in equilibrium and on the point of sliding down the plane.
Brick $P$ is modelled as a particle.
Using the model,
(a) find, in terms of $m$ and $g$, the magnitude of the normal reaction of the plane on brick $P$
(b) show that $\mu=\frac{3}{4}$

## For parts (c) and (d), you are not required to do any further calculations.

Brick $P$ is now removed from the plane and a much heavier brick $Q$ is placed on the plane.

The coefficient of friction between $Q$ and the plane is also $\frac{3}{4}$
(c) Explain briefly why brick $Q$ will remain at rest on the plane.

Brick $Q$ is now projected with speed $0.5 \mathrm{~m} \mathrm{~s}^{-1}$ down a line of greatest slope of the plane.
Brick $Q$ is modelled as a particle.
Using the model,
(d) describe the motion of brick $Q$, giving a reason for your answer.

2 A particle $P$ moves with acceleration $(4 \mathbf{i}-5 \mathbf{j}) \mathrm{m} \mathrm{s}^{-2}$
At time $t=0, P$ is moving with velocity $(-2 \mathbf{i}+2 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$
(a) Find the velocity of $P$ at time $t=2$ seconds.

At time $t=0, P$ passes through the origin $O$.
At time $t=T$ seconds, where $T>0$, the particle $P$ passes through the point $A$.
The position vector of $A$ is $(\lambda \mathbf{i}-4.5 \mathbf{j}) \mathrm{m}$ relative to $O$, where $\lambda$ is a constant.
(b) Find the value of $T$.
(c) Hence find the value of $\lambda$

3 (i) At time $t$ seconds, where $t \geq 0$, a particle $P$ moves so that its acceleration a m s${ }^{-2}$ is given by

$$
\mathbf{a}=(1-4 t) \mathbf{i}+\left(3-t^{2}\right) \mathbf{j}
$$

At the instant when $t=0$, the velocity of $P$ is $36 \mathbf{i} \mathrm{~m} \mathrm{~s}^{-1}$
(a) Find the velocity of $P$ when $t=4$
(b) Find the value of $t$ at the instant when $P$ is moving in a direction perpendicular to $\mathbf{i}$
(ii) At time $t$ seconds, where $t \geq 0$, a particle $Q$ moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin $O$, is given by

$$
\mathbf{r}=\left(t^{2}-t\right) \mathbf{i}+3 t \mathbf{j}
$$

Find the value of $t$ at the instant when the speed of $Q$ is $5 \mathrm{~m} \mathrm{~s}^{-1}$


Figure 1
A ladder $A B$ has mass $M$ and length $6 a$.
The end $A$ of the ladder is on rough horizontal ground.
The ladder rests against a fixed smooth horizontal rail at the point $C$.
The point $C$ is at a vertical height $4 a$ above the ground.
The vertical plane containing $A B$ is perpendicular to the rail.
The ladder is inclined to the horizontal at an angle $\alpha$, where $\sin \alpha=\frac{4}{5}$ as shown in Figure 1 .

The coefficient of friction between the ladder and the ground is $\mu$.
The ladder rests in limiting equilibrium.
The ladder is modelled as a uniform rod.
Using the model,
(a) show that the magnitude of the force exerted on the ladder by the rail at $C$ is $\frac{9 M g}{25}$
(b) Hence, or otherwise, find the value of $\mu$.


Figure 2

A small ball is projected with speed $U \mathrm{~m} \mathrm{~s}^{-1}$ from a point $O$ at the top of a vertical cliff.
The point $O$ is 25 m vertically above the point $N$ which is on horizontal ground.
The ball is projected at an angle of $45^{\circ}$ above the horizontal.
The ball hits the ground at a point $A$, where $A N=100 \mathrm{~m}$, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.
Using this initial model,
(a) show that $U=28$
(b) find the greatest height of the ball above the horizontal ground $N A$.

In a refinement to the model of the motion of the ball from $O$ to $A$, the effect of air resistance is included.

This refined model is used to find a new value of $U$.
(c) How would this new value of $U$ compare with 28 , the value given in part (a)?
(d) State one further refinement to the model that would make the model more realistic.

October 2020 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here - they will be covered in the formal mark scheme.

This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2020.

Guidance on the use of codes within this document

M1 - method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.

A1 - accuracy mark. This mark is generally given for a correct answer following correct working.

B1 - working mark. This mark is usually given when working and the answer cannot easily be separated.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

## Question 1 (Total 9 marks)

| Part | Working or answer an examiner might expect to see | Mark | Notes |
| :---: | :---: | :---: | :---: |
| (a) | $R=m g \cos \alpha=\frac{4}{5} m g$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for a method to resolve perpendicular to the plane |
|  |  | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression for $R$ |
| (b) | $F=m g \sin \alpha$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for a method to resolve parallel to the plane |
|  |  | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression for $F$ |
|  | $F=\mu R$ | $\begin{aligned} & \text { M1 } \\ & 2.1 \end{aligned}$ | This mark is given for a method to use $F=\mu R$ to solve for $\mu$ |
|  | $\mu=\frac{3}{4}$ | $\begin{gathered} \mathrm{A} 1 \\ \text { 2.2a } \end{gathered}$ | The mark is given for fully correct working leading to the given answer |
| (c) | The forces acting on $Q$ will still balance as the $m$ terms cancel | $\begin{aligned} & \text { B1 } \\ & 2.4 \end{aligned}$ | This mark is given for a correct explanation |
| (d) | $Q$ slides down the plane with constant speed $0.5 \mathrm{~m} \mathrm{~s}^{-1}$ | $\begin{aligned} & \text { B1 } \\ & 2.4 \end{aligned}$ | This mark is given for a correct description of the constant speed of $Q$ |
|  | There is no resultant force down the plane so there is no acceleration | $\begin{aligned} & \text { B1 } \\ & 2.4 \end{aligned}$ | This mark is given for a correct description of acceleration |

## Question 2 (Total 8 marks)

| Part | Working or answer an examiner might expect to see | Mark | Notes |
| :---: | :---: | :---: | :---: |
| (a) | $\mathbf{v}=(-2 \mathbf{i}+2 \mathbf{j})+2(4 \mathbf{i}-5 \mathbf{j})$ | $\begin{aligned} & \text { M1 } \\ & \text { 3.1a } \end{aligned}$ | This mark is given for a method to use $\mathbf{v}=\mathbf{u}+\mathbf{a} t$ |
|  | $=6 \mathbf{i}-8 \mathbf{j} \mathrm{~m} \mathrm{~s}^{-1}$ | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct answer only |
| (b) | $(\lambda \mathbf{i}-4.5 \mathbf{j})=(-2 \mathbf{i}+2 \mathbf{j}) t+\frac{1}{2} t^{2}(4 \mathbf{i}-5 \mathbf{j})$ | $\begin{aligned} & \text { M1 } \\ & \text { 3.1a } \end{aligned}$ | This mark is given for a method to use $\mathbf{r}=\mathbf{u} t+\frac{1}{2} \mathbf{a} t^{2}$ |
|  |  | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression for $\mathbf{r}$ |
|  | $-4.5=2 T-\frac{5}{2} T^{2}$ | $\begin{aligned} & \text { M1 } \\ & 2.1 \end{aligned}$ | This mark is given for a method to attempt to equate $\mathbf{j}$ components to give an equation in $T$ only |
|  | $T=1.8$ | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct answer only |
| (c) | $-2 \times 1.8+\frac{1}{2}(1.8)^{2} \times 4$ | $\begin{aligned} & \text { M1 } \\ & 3.1 \mathrm{a} \end{aligned}$ | This mark is given for substituting the value of T into the $\mathbf{i}$ component to find a value for $\lambda$ |
|  | $\lambda=2.88$ | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct answer only |

## Question 3 (Total 12 marks)

| Part | Working or answer an examiner might expect to see | Mark | Notes |
| :---: | :---: | :---: | :---: |
| (a) | $\mathbf{v}=\left(t-2 t^{2}\right) \mathbf{i}+\left(3 t-\frac{1}{3} t^{3}\right) \mathbf{j}(+\mathbf{c})$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for a method to integrate a to find the velocity $\mathbf{v}$ |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression for the velocity $\mathbf{v}$ |
|  | When $t=4$, $\mathbf{v}=8 \mathbf{i}-\frac{28}{3} \mathbf{j} \mathrm{~m} \mathrm{~s}^{-1}$ | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct answer only |
| (b)(i) | $t-2 t^{2}+36=0$ | $\begin{aligned} & \text { M1 } \\ & \text { 3.1a } \end{aligned}$ | This mark is given for a method to equate the $\mathbf{i}$ component of $\mathbf{v}$ to zero |
|  |  | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression in terms of $t$ |
|  | $t=4.5$ | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct answer only |
| (b)(ii) | $\mathbf{v}=(2 t-1) \mathbf{i}+3 \mathbf{j}$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for a method to differentiate $\mathbf{r}$ to find the velocity $\mathbf{v}$ |
|  |  | $\begin{aligned} & \text { A1 } \\ & 1.1 \mathrm{~b} \end{aligned}$ | This mark is given for a correct expression for $\mathbf{v}$ |
|  | $(2 t-1)^{2}+3^{2}=5^{2}$ | $\begin{aligned} & \text { M1 } \\ & 2.1 \end{aligned}$ | This mark is given for a method to use the magnitude to find an expression in terms of $t$ |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression in terms $\mathrm{f} t$ |
|  | $\begin{aligned} & 4 t^{2}-4 t+1+9=25 \\ & 4 t^{2}-4 t-15=0 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { 3.1a } \end{aligned}$ | This mark is given for a method to solve an equation for $t$ |
|  | $t=2.5$ | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct answer only |

## Question 4 (Total 10 marks)

| Part | Working or answer an examiner might expect to see | Mark | Notes |
| :---: | :---: | :---: | :---: |
| (a) | $N \times \frac{4 a}{\sin \alpha}=M g \times 3 a \cos \alpha$ | $\begin{aligned} & \text { M1 } \\ & 3.3 \end{aligned}$ | This mark is given for a method to take moments about $A$ |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for correctly taking moments about $A$ |
|  | $\frac{9 M g}{25}$ | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | The mark is given for fully correct working leading to the given answer |
| (b) | $(\rightarrow) F=\frac{9 M g}{25} \sin \alpha$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for a method to resolve horizontally |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression for $(\rightarrow) F$ |
|  | $(\uparrow) R+\frac{9 M g}{25} \cos \alpha=M g$ | $\begin{gathered} \text { M1 } \\ 3.4 \end{gathered}$ | This mark is given for a method to resolve vertically |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct expression for $(\uparrow) R$ |
|  | $F=\mu R$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for using $F=\mu R$ |
|  | $\begin{aligned} & \frac{9 M g}{25} \times \frac{4}{5}=\mu\left(M g-\frac{9 M g}{25} \times \frac{3}{5}\right) \\ & 36 M g=\mu \times 98 M g \end{aligned}$ | $\begin{gathered} \text { M1 } \\ 3.1 \mathrm{~b} \end{gathered}$ | This mark is given for a method to eliminate $R$ and $F$ to solve for $\mu$ |
|  | $\mu=\frac{18}{49}$ | $\begin{gathered} \text { A1 } \\ 2.2 \mathrm{a} \end{gathered}$ | This mark is given for a correct answer only |

## Question 5 (Total 11 marks)

| Part | Working or answer an examiner might expect to see | Mark | Notes |
| :---: | :---: | :---: | :---: |
| (a) | $U \cos 45^{\circ} t=100$ | $\begin{aligned} & \mathrm{M} 1 \\ & 3.3 \end{aligned}$ | This mark is given for a method to use horizontal motion |
|  |  | $\begin{aligned} & \text { M1 } \\ & 1.1 \mathrm{~b} \end{aligned}$ | This mark is given for a correct equation in $U$ and $t$ |
|  | $U \sin 45^{\circ} t-\frac{1}{2} g t^{2}=-25$ | $\begin{aligned} & \mathrm{M} 1 \\ & 3.4 \end{aligned}$ | This mark is given for a method to use vertical motion |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for a correct equation in $U$ and $t$ |
|  | $\begin{aligned} & t=\frac{100 \sqrt{ } 2}{U} \\ & \frac{U}{\sqrt{ } 2} \times \frac{100 \sqrt{ } 2}{U}-\frac{1}{2} g\left(\frac{100 \sqrt{ } 2}{U}\right)^{2}=-25 \\ & 100-4.9 \times \frac{20000}{U^{2}}=-25 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & 3.1 \mathrm{~b} \end{aligned}$ | This mark is given for a method to eliminate $t$ and solve for $U$ |
|  | $U=28$ | $\begin{gathered} \text { A1 } \\ 1.1 \mathrm{~b} \end{gathered}$ | The mark is given for fully correct working leading to the given answer |
| (b) | $0^{2}=\left(28 \sin 45^{\circ}\right)^{2}-2 g h$ | $\begin{aligned} & \text { M1 } \\ & 3.4 \end{aligned}$ | This mark is given for a method to use vertical motion |
|  |  | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for finding a correct equation in terms of $h$ only |
|  | Greatest height $=45 \mathrm{~m}$ | $\begin{gathered} \mathrm{A} 1 \\ 1.1 \mathrm{~b} \end{gathered}$ | This mark is given for the correct answer only |
| (c) | The new value is greater than 28 | $\begin{gathered} \text { B1 } \\ 3.5 \mathrm{a} \end{gathered}$ | This mark is given for a correct statement |
| (d) | For example: <br> Account for the effects of wind <br> Use a more accurate value for $g$ <br> Account for the spin of the ball | $\begin{gathered} \text { B1 } \\ 3.5 \mathrm{c} \end{gathered}$ | This mark is given for a valid refinement |

